

# Carbon dioxide removal and directed technical change

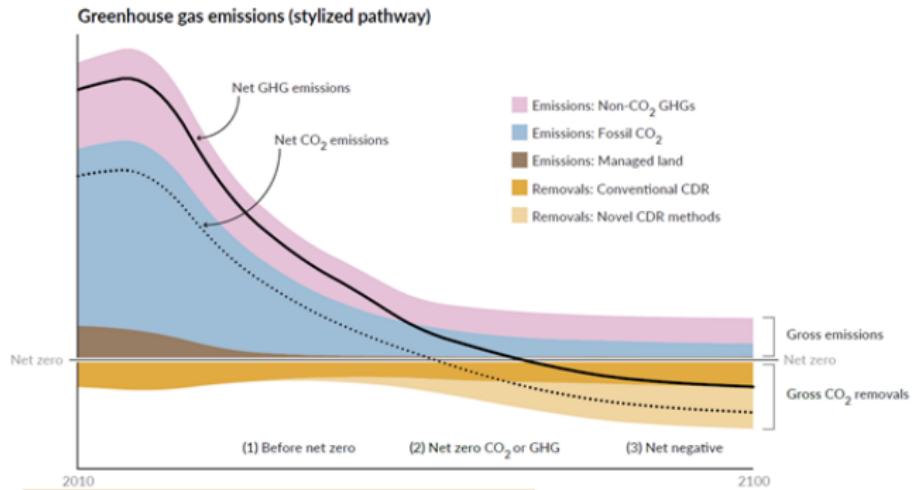
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# Introduction

- ▶ Climate goals require large-scale deployment of Carbon Dioxide Removal (CDR)
- ▶ CDR technologies are currently too expensive to be competitive → innovation would help
- ▶ How to incentivize the green transition when CDR is necessary and innovation is endogenous?

# Residual emissions



**Figure 1.1** Roles of carbon dioxide removal (CDR) in ambitious mitigation strategies, applicable at national and global levels. Basic emission and removal components of mitigation pathways, and the corresponding trajectories for both net carbon dioxide (CO<sub>2</sub>) and greenhouse gas (GHG) emissions. (Adapted from Babiker et al., 2022).<sup>4</sup>

Source: Smith et al. (2024)

# CDR is expensive

CDR method	Weighted average price (\$)	
	2022	2023
Afforestation/reforestation	12	16
Bioenergy with carbon capture and storage	No data	300
Biochar	212	131
Biomass burial	92	111
Bio-oil storage	600	505
Direct air carbon capture and storage	1,261	715
Direct ocean carbon capture and storage	984	1,402
Enhanced rock weathering	434	371
Forest management	15	12
Mineral products	471	No data
Ocean alkalinity enhancement	No data	1,608
<b>Total</b>	<b>303</b>	<b>488</b>

**Table 4.2** Volume-weighted average price per carbon credit from transactions where the price is known, by carbon dioxide removal (CDR) method, 2022–2023. Durable wood products are not included. Data sources: CDR.fyi, 2024;<sup>125</sup> Ecosystem Marketplace, 2023.<sup>126</sup>

Source: Smith et al. (2024)

## The policy questions

How to optimally fit CDR deployment and innovation policies into the green transition?

- ▶ Should we integrate markets for positive and negative emissions?
- ▶ Should governments focus on deployment or innovation?
- ▶ Do CDR policies slow down the fossil phase-out?
- ▶ Can "overshooting" be optimal?

## Literature

- ▶ Growing literature on CDR (Edenhofer et al., 2025)
  - ▶ Stressing the need for deployment to bring down costs of CDR (e.g., Nemet et al., 2023)
  - ▶ Experience curve estimation: Sievert et al. (2024)
  - ▶ Warning about mitigation deterrence (McLaren, 2020)
  - ▶ Integrating CDR into carbon markets (e.g., Sultani et al., 2024; Verbist et al., 2025)
- ▶ We build on literature on directed technical change and the environment
  - ▶ Clean-dirty: Smulders and de Nooij (2003); Acemoglu et al. (2012); Smulders and Zhou (forthcoming AEJmacro)
  - ▶ Three directions: Durmaz and Schroyen (2020); Acemoglu et al. (2023); Alsina-Pujols and Hovdahl (2024); Gentile (2024)

## Research contributions

- ▶ Extending DTC model (Acemoglu et al., 2023)
  - ▶ CDR as an endogenous innovation sector
  - ▶ Accounting for unabatable emissions
- ▶ Analytical results on optimal CDR deployment and temperature overshoot
- ▶ Quantitative results on optimal and second-best policies and technology/price shocks

# The model

## Preferences and final good

Following the structure of Acemoglu et al. (2023):

- ▶ Preferences:

$$U_t = \sum_{s=t}^{\infty} \frac{1}{(1+\rho)^{s-t}} \log C_s$$

- ▶ Final good production:

$$Y_t = \left( (1-\nu)(A_{Pt}L_{Pt})^{\frac{\sigma-1}{\sigma}} + \nu E^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Adjusted version of Dietz and Venmans (2019):

- ▶ Temperature is linear in cumulative CO<sub>2</sub> emissions:

$$T_t = S_t$$

- ▶ Abatable emissions, residual emissions and removals:

$$S_t = S_{t-1} + E_{d,t-1} + E_{u,t-1} - R_{t-1}$$

## Energy and removal

- ▶ Energy composite consists of clean and dirty energy (substitutes):

$$E_t = \left( \kappa E_{ct}^{\frac{\epsilon-1}{\epsilon}} + (1 - \kappa) E_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

- ▶ Clean and dirty energy and removals:

$$E_{it} = \exp \left( \int_0^1 \ln A_{ijt} l_{ijt} dj \right), \quad R_t = \exp \left( \int_0^1 \ln A_{rjt} l_{rjt} dj \right)$$

- ▶ All sectors use labor (Riccardian)

$$L_{Pt} + L_{ct} + L_{dt} + L_{rt} = 1$$

## Productivity and innovation

- ▶ Average productivity  $i \in \{c, d, r\}$  is

$$A_{it} = \exp \left( \int_0^1 \ln A_{ijt} dj \right)$$

- ▶ Innovation effort  $s$  leads to productivity growth at rate:

$$\ln(A_{it}/A_{it-1}) = (\ln \gamma) \eta_i s_{it}^{1-\psi}$$

- ▶ step size  $\gamma$
- ▶ R&D productivity  $\eta_i$
- ▶ stepping on toes  $\psi$
- ▶ Productivity in intermediates  $A_P$  grows exogenously
- ▶ Given supply of scientists:

$$s_{ct} + s_{dt} + s_{rt} = 1$$

# Equilibrium

## Market:

- ▶ Monopolistic competition in production – constant markup  $\gamma$ .
- ▶ Successful innovators receive monopoly rights for one period.
- ▶ Free entry in R&D – return equalized across sectors  
 $i = c, d, r$ .

## Government:

- ▶ Environmental policy: pollution tax or permits.
- ▶ CDR deployment policy: credits (integrated permit market) or direct subsidy
- ▶ R&D policy: subsidy to R&D.

# Analytical results

## Main analytical questions

- ▶ When start CDR deployment?
- ▶ When start CDR research?
- ▶ Overshoot?

## When start CDR deployment?

Cost effective mix mitigation and removal:  
start only when net emission target sufficiently stringent.

- ▶ First unit of CDR is costly
- ▶ First unit of mitigation (replace fossil by clean) is costless.

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Let  $M$  be net emissions and  $(\partial Y / \partial E_c) A_c = w$  be wage

- ▶ If  $M$  is not stringent  $\partial Y / \partial E_d - w / A_d < w / A_r$ ,  
then only mitigation
- ▶ If  $M$  is stringent  $\partial Y / \partial E_d - w / A_d = w / A_r$ ,  
then both mitigation and CDR  
 $\Rightarrow MAC = \partial Y / \partial M$  constant!

## When start CDR deployment?

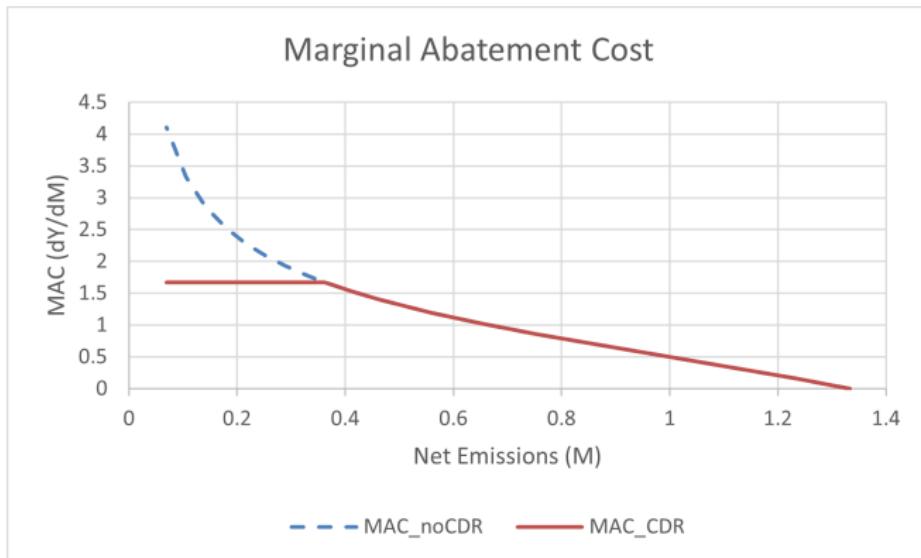
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 $\Rightarrow MAC = \partial Y / \partial M$  constant!

Cost-effectiveness arises in integrated permit market:  
emission permit price = removal credit



## Why MAC becomes constant under sufficient stringency

Start at Net Zero economy  $M = 0$ :

- ▶ Ricardian economy (labor is only production factor)
- ▶ cost of **fossil** energy includes full carbon removal  $\Rightarrow$  cost is  $1/A_d + 1/A_r \equiv 1/A_f$  units of labor.
- ▶ Static labor allocation reduces to standard **three**-sector Ricardian economy with productivities,  $A_P, A_c, A_f$ .

Now allow the economy to generate  $M > 0$  emissions.

- ▶ This frees up  $M/A_r$  units of labor in the CDR sector
- ▶ More labor available for production sectors  $d, c, P$
- ▶ Because of CRS with respect to  $d, c, P$  in production, relative labor allocation and relative productivity remains unchanged
- ▶ Hence same expansion of output for each unit of emissions, i.e. constant  $MAC \equiv \partial Y / \partial M$ .

Once  $M > M^{\#}$ , all CDR labor is already reallocated and emissions can be only increased by dirty/clean substitution. Now MAC falls with emissions.

## How quickly introduce CDR?

Paris style policy:

- ▶ fix target date  $t^\#$  and temperature target  $T^\#$

Implies **carbon budget**  $T^\# - T_0 \equiv B$

- ▶ assume the budget is binding:  $B < \int_0^{t^\#} M^{LF}(t)dt$

Cost-effectiveness: minimize NPV of cost of reaching target.

- ▶ then, until time  $t^\#$ , **MAC must grow at rate  $r$** ,
- ▶ where  $r = \rho + \hat{Y}$  is the consumption discount rate

# How quickly introduce CDR?

Benchmark: cost-effectiveness when **technology is constant**.

- ▶ MAC declines with  $M$  if no CDR is deployed
- ▶ MAC constant if CDR is deployed

Under cost-effectiveness  $MAC$  must grow at rate  $r$ :

- ▶ No CDR until  $t^\#$
- ▶ Net emissions fall over time, jump to zero at time  $t^\#$ .
- ▶ Temperature might overshoot:
  - ▶ either temperature overshoots and negative emissions at target date (if target date  $t^\#$  far away and target temperature  $T^\#$  stringent),...
  - ▶ ...or ample carbon budget to postpone net zero and never have negative emissions.

Proof in Appendix

## How quickly introduce CDR?

Beyond benchmark: what if **technology changes**?

- ▶ The three technology levels have different impacts on MAC
  - ▶  $A_d$  increases MAC
  - ▶  $A_r$  reduces MAC under CDR
  - ▶  $A_c$  increases MAC under CDR (!)
- ▶ Now possible: early CDR and no overshoot
  - ▶ CDR starts before target date and MAC grows at rate  $r$ ;
  - ▶ R&D provides investment opportunity  $\Rightarrow$  smooth consumption  
 $\Rightarrow$  avoid overshoot and negative emissions

## When start CDR research?

- ▶ Cost of developing a patent increases with effort /research intensity, first unit free
- ▶ Market: if patents last only one period, current market size matters  $\Rightarrow$  R&D only starts with deployment
- ▶ Optimum: net present value matters + adjustment costs  $\Rightarrow$  start immediately

# Quantitative results

# Calibration - data

**Table:** Data to be matched in calibration

Variable	Value	Sources and notes
World emissions	37.79 GtCO <sub>2</sub>	Friedlingstein et al. (2024)
Temperature increase	1.1 °C	IPCC (2023)
Residual emissions	6.5 GtCO <sub>2</sub>	Smith et al. (2024, p. 150)
Global carbon price	5.56 \$ per tCO <sub>2</sub>	Dolphin and Merkle (2024)
Energy share of GDP	9%	Agnolucci et al. (2025)
$R_0$	1.35 MtCO <sub>2</sub>	Smith et al. (2024, p. 124)
$p_{r0}$	488 \$ per tCO <sub>2</sub>	Smith et al. (2024, p. 81)
$E_{c0}$	30,035 TWh	IEA (2024)
$E_{d0}$	142,441 TWh	IEA (2024)
$p_{c0}$	59.92 \$ per MWh	Own calculations based on
$p_{d0}$	54.45 \$ per MWh	Lazard (2024); IRENA (2025)
$Y_0$	106.17 trillion current \$	World Bank

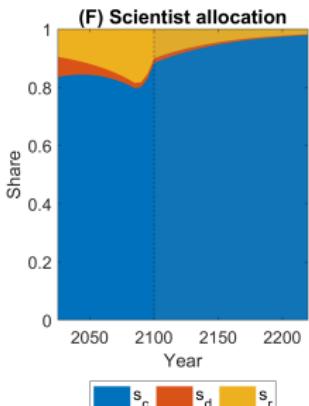
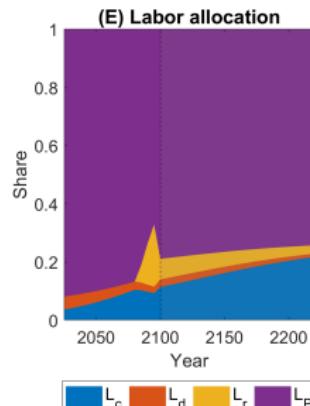
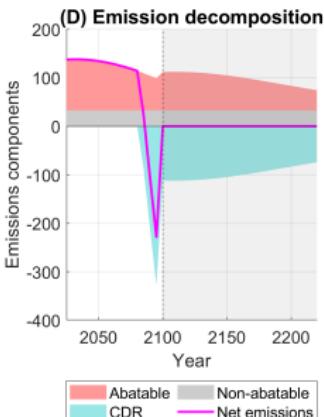
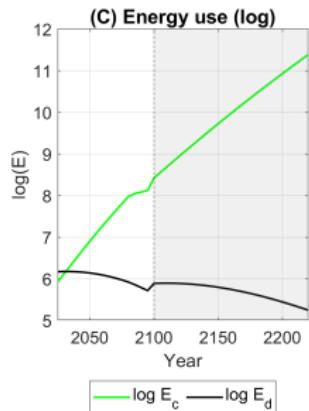
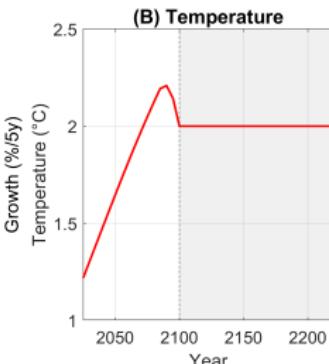
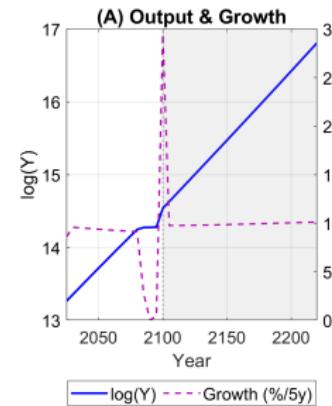
Notes: All quantities in this table are per year. They are multiplied by five in the calibration.

## Calibration - parameters

Table: Parameter values and initial technology stocks

Parameter	Value	Sources and notes
$\sigma$	0.4	Acemoglu et al. (2023); van der Werf (2008)
$\nu$	0.5	Normalization
$\varepsilon$	1.8561	Acemoglu et al. (2023); Papageorgiou et al. (2017)
$\Psi$	32.5	Set by us based on Smith et al. (2024)
$\kappa$	0.3166	Match clean and dirty energy prices and quantities
$\gamma$	1.07	Acemoglu et al. (2023)
$\eta$	1.4634	Acemoglu et al. (2023)
$\psi$	0.5	Acemoglu et al. (2023); Blundell et al. (2002)
$\rho$	0.01 per year	Acemoglu et al. (2023)
$\xi$	$0.000480 \times 1.3$	Dietz and Venmans (2019) and own decision
$\phi_d$	0.2197	Match abatable emissions to dirty energy cons.
$\phi_r$	1/1.3	Own decision
$\tilde{A}_E$	3,381.64	Calibrated
$A_{c0}$	9,405.22	Calibrated
$A_{d0}$	10,350.05	Calibrated
$A_{r0}$	1,154.80	Calibrated
$A_{P0}$	389,671.96	Calibrated

# Social optimum



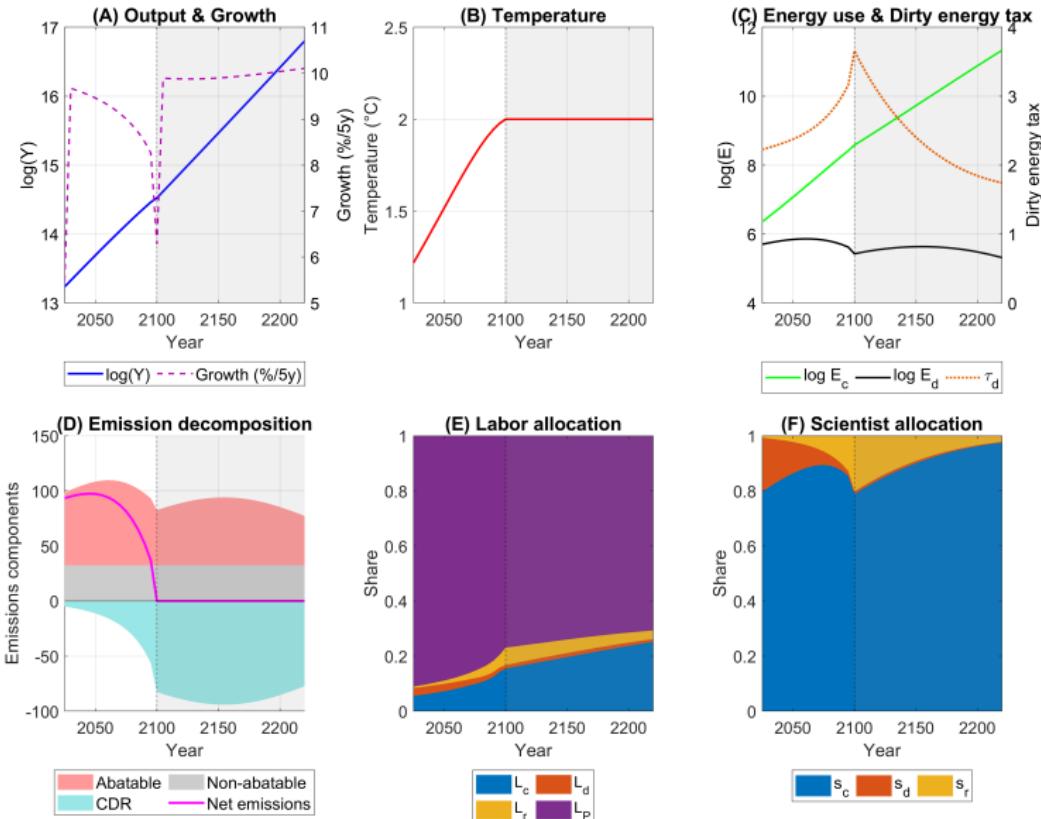
## Social optimum

- ▶ No deployment of CDR until 2085
- ▶ Large R&D investments in CDR during the transition
- ▶ Massive reallocation of labor at the end of the transition
- ▶ Overshoot is optimal
- ▶ Dirty energy is not fully phased out

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- ▶ Large R&D investments in CDR during the transition
- ▶ Massive reallocation of labor at the end of the transition
- ▶ Overshoot is optimal
- ▶ Dirty energy is not fully phased out
- ▶ Can be implemented with tax on dirty energy, budget for CDR, subsidies for clean and removal R&D
  - ▶ Or, instead of tax and budget: integrated carbon market with carbon budget for 2° and banking until 2100, followed by annual cap of 0
  - ▶ Research subsidy for R&D cannot be a percentage on top of profits; must be independent of deployment

# Second best: no innovation subsidies



## Second best

- ▶ Deployment is necessary to stimulate innovation
  - ▶ Even though price of removal > price of reduction
  - ▶ Looks small but is instant 960-fold increase in deployment (16,000-fold by 2100)
- ▶ Overshoot no longer optimal
  - ▶ Emissions are reduced more than in the optimum
  - ▶ Carbon tax increases, then decreases
- ▶ More dirty R&D taking place than in the optimum
  - ▶ Increasing future MAC
  - ▶ CDR research starts up too slowly

## Integrated carbon market

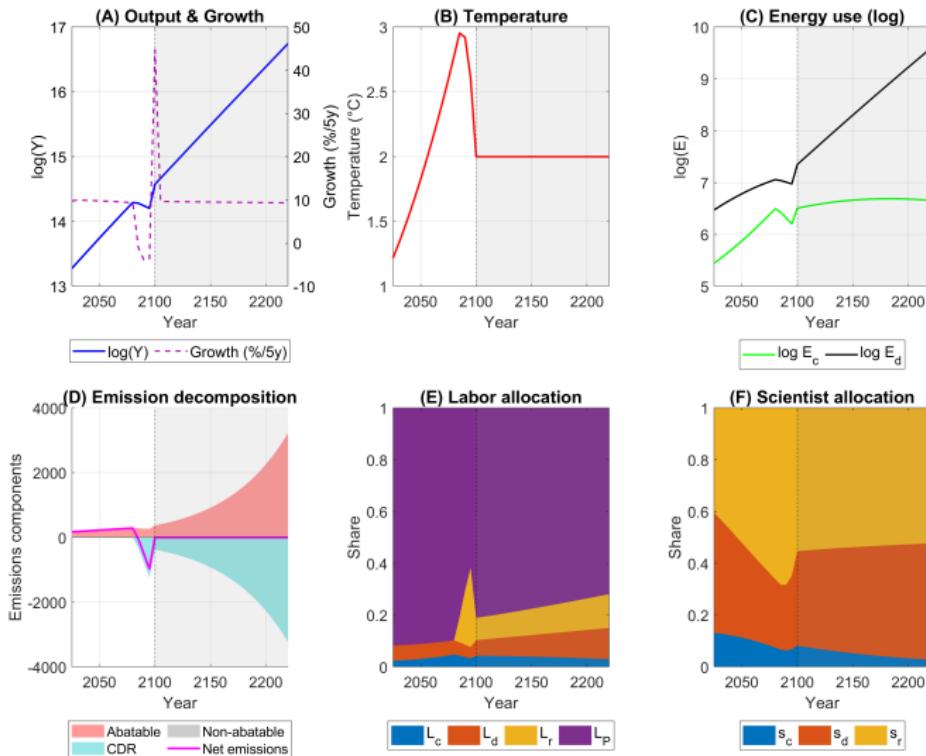
If R&D subsidies independent of production are not feasible:

- ▶ Integrated carbon market is a bad idea
- ▶ Deployment in an integrated market starts too late if CDR tech is far behind
- ▶ No deployment  $\Rightarrow$  no R&D  $\Rightarrow$  no cost reduction

Better to subsidize deployment early, independent of ETS

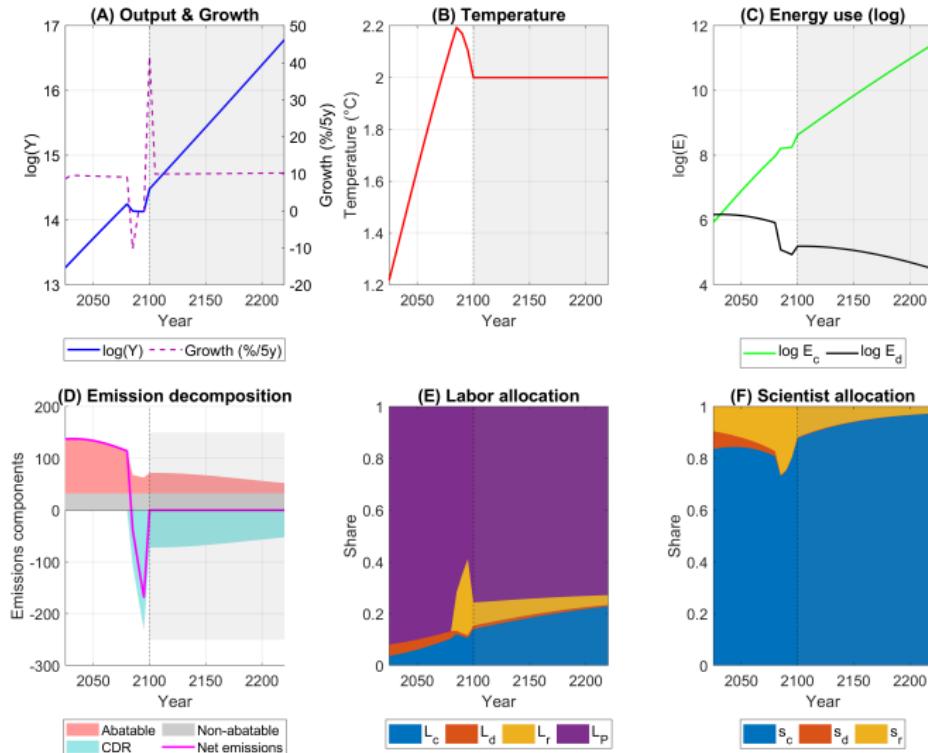
# Can CDR push us to a "dirty" equilibrium?

Yes! If CDR (suddenly) becomes more productive (e.g.,  $A_{r0} \times 1.5$ ):



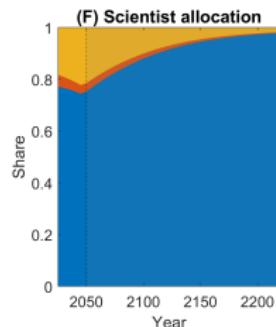
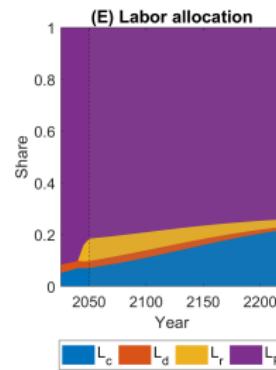
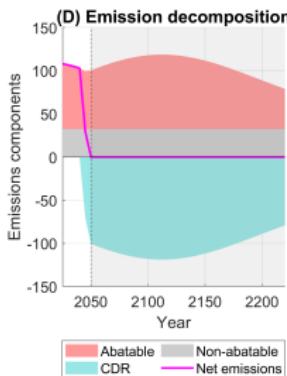
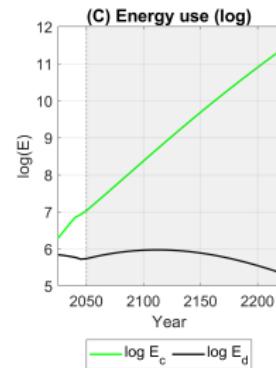
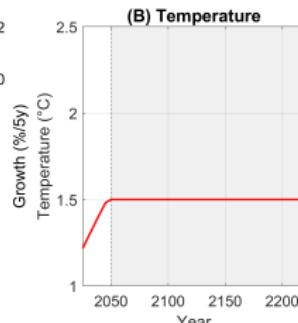
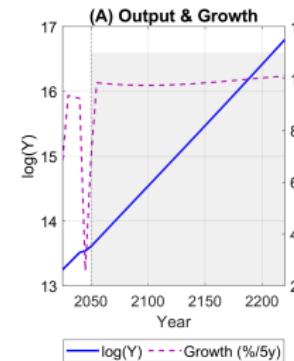
# What if CDR disappoints when we need it? ( $A_{r,2085} \times 0.5$ )

The transition becomes much more costly:



# Finally, what if our goal is $1.5^{\circ}$ in 2050?

No overshoot as CDR is too costly



# Conclusions

## Conclusions

We built a model of endogenous innovation with CDR and residual emissions

- ▶ Optimal policy requires massive innovation investments in CDR but postponing deployment
- ▶ "Realistic" policy combines CDR deployment with emission reduction
- ▶ Integrating positive and negative emission markets risks a failure to incentivize innovation

Thanks!

Any questions or suggestions?

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## Proof overshoot [SKIP IN PRESENTATION]

Define  $m(t)$ , for  $0 < t < t^\#$ , such that if  $M(t) = m(t)$ , MAC grows at  $r$  and reaches  $M^\#$  (i.e. activation level of CDR).

[assume this path exists – requires MAC smooth around LF level (“first unit of abatement is costless”)]

Define  $\theta = \int_0^{t^\#} m(t)dt$ , cumulative emissions (i.e. temperature change just before target date).

- ▶ If  $\theta = B$  then  $M$  jumps to zero at  $t^\#$ .
- ▶ If  $\theta > B$  then negative emissions needed: “overshoot”
- ▶ If  $t^\# M^{LF} < B$  then all action (mitigation and CDR) postponed till after  $t^\#$  (before carbon budget cannot be used productively). Not realistic case: carbon budget does not bind.
- ▶ If  $\theta < B$  then no overshoot, i.e. remainder carbon budget is used after  $t^\#$  until  $t^a$ .

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# Proof overshoot

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[assume this path exists – requires MAC smooth around LF level (“first unit of abatement is costless”)]

Define  $\theta = \int_0^{t^\#} m(t)dt$ , cumulative emissions (i.e. temperature change just before target date).

- ▶ If  $\theta = B$  then  $M$  jumps to zero at  $t^\#$ .
- ▶ If  $\theta > B$  then negative emissions needed: “overshoot”
- ▶ If  $t^\# M^{LF} < B$  then all action (mitigation and CDR) postponed till after  $t^\#$  (before carbon budget cannot be used productively). Not realistic case: carbon budget does not bind.
- ▶ If  $\theta < B$  then no overshoot, i.e. remainder carbon budget is used after  $t^\#$  until  $t^a$ .

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# APPENDIX: Short-sighted Innovation

AABH-style one-period-patents.

Relative R&D costs equal relative profits:

$$\frac{\eta_k}{\eta_j} \left( \frac{s_j}{s_k} \right)^\psi = \left( \frac{p_j A_j L_j}{p_k A_k L_k} \right) = \left( \frac{A_i / (1 + \tau_i)}{A_i / (1 + \tau_i)} \right)^{\epsilon-1}$$

TFP growth:

$$\hat{A} = \left( \frac{\left[ \sum_i \eta_i^{1/\psi} \left( \frac{A_i}{1 + \tau_i} \right)^{(\epsilon-1)/\psi} \right]^{\psi/(\epsilon-1)}}{\left[ \sum_i \left( \frac{A_i}{1 + \tau_i} \right)^{\epsilon-1} \right]^{1/(\epsilon-1)}} \right)^{\epsilon-1} (\ln \gamma) s^{1-\psi}$$

## APPENDIX Forward-looking Innovation

Value of a patent,  $v_i$ , is Net Present Value of profits.

Relative R&D costs equal relative patent values:

$$\frac{\eta_k}{\eta_j} \left( \frac{s_j}{s_k} \right)^\psi = \frac{v_j}{v_k}$$

$$rv_i = \gamma p_i L_i + \dot{v}_i$$

Coordination failure possible (if  $\epsilon$  large).

- ▶ fossil path: everybody believes fossil stays – invest in  $d$  and  $r$  – fossil cheap – demand high – beliefs justified
- ▶ green path: everybody believes fossil stops – invest in  $c$  – clean cheap – demand high – beliefs justified

## APPENDIX Optimal Innovation

Value of a patent,  $\lambda_i$ , is Net Present Value of contributions to welfare (including spillovers and constraint relaxing).

Relative R&D costs equal relative social values:

$$\frac{\eta_k}{\eta_j} \left( \frac{s_j}{s_k} \right)^\psi = \frac{\lambda_j A_j}{\lambda_k A_k}$$

$$r\lambda_i A_i = p_i Y_i / Y + \frac{d}{dt} \lambda_i A_i$$