Public Goods, Optimal Taxation, and Heterogeneity

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Distributional aspects are essential for environmental policy for many reasons (e.g., Drupp et al. 2025, *JEL*)

This paper deals with optimal provision of public goods, such as environmental public goods (and bads), in the presence of optimal non-linear income taxation

How to deal with distribution in cost-benefit analysis is a far from novel topic

On the one hand, “a dollar is a dollar”

On the other hand, for the government, a dollar for the poor is typically valued higher than for the rich; then maybe a dollar in WTP should also be valued higher for the poor?

On the third hand, also distributional policies should presumably be conducted cost-effectively, and there are different ways to deal with equity issues

The Atkinson and Stiglitz (1976, *JPubE*) theorem:

Given optimal non-linear income taxation and weak separability between consumption of the different good, on the one hand, and labor of the other, it is optimal to have the same consumption tax rate for all goods

Thus, one should not have lower consumption taxes for good primarily consumed by the poor

Instead, distributional concerns should be handled solely through income taxation

Hylland and Zeckhauser (1979, *SJE*) was the first study deriving optimal public good provision rules under optimal non-linear income taxes in a paper with the informative title:

"Distributional Objectives Should Affect Taxes but Not Program Choice or Design."

They identified conditions for when the basic Samuelson rule applies, i.e., simply equalizing aggregate *MRS* with *MRT*, without taking any distributional concerns

Christiansen (1981, *RES*) generalized these results to conclude that if all individuals have identical weakly separable preferences

$$u\left(v\left(c,G\right),l\right)$$

where c is consumption, G is the amount of the public good, and l is labor, then the basic Samuelson rule applies

Kaplow (2024, *JEL*) and others conclude that this is the most reasonable benchmark case that should guide public policy

Thus, the first-best Samuelson rule should then guide provision of public goods, and cost-benefit analysis more generally

Clearly, strictly speaking, people do not have identical preferences

Nor is there any particular reason to believe that this separability structure applies

Still, we will always have to make simplifying assumptions

As an approximation, how bad can the Christiansen assumptions be?

Very, very bad!

The problem is not the weak separability part

The main problem is the identical utility function part!

By assuming a common utility function

$$u\big(v(c,G),l\big)$$

we are not only assuming identical *preferences*

We are also implicitly assuming identical *exposure* to the public good (or bad)!

But in reality, exposure can of vary substantially, and also systematically related to income

Surprisingly, no previous study has incorporated public good in the optimal taxation problem using the perturbation (sometimes denoted sufficient statistics) approach to optimal taxation (Saez 2001, *RES*)

The perturbation approach is superior to the mechanism design approach (Mirrlees 1971, *RES*) in analyzing heterogeneity

The model

People have preferences increasing in consumption c, decreasing in before-tax income z, and increasing in the public good G

$$u(c, z, G; \psi)$$

ψ is a broad multi-dimensional indicator variable that can reflect all kinds of preference heterogeneity, exposure heterogeneity, and also various kinds of ability heterogeneity

The government cannot observe ψ

The policy instruments available consist of a non-linear income tax $T(z)$ and the provision of a public good, G

An individual with income z will then have a marginal willingness to pay (*MWTP*) for a public good increase that also depends on his/her type ψ :

$$MRS_{Gc}(c(z), z, G, \psi)$$

We assume that the *MRS* can be observed by the government

(as is standard)

The governmental objective: a generalized utilitarian SWF

$$W = \int_0^{\infty} \int_{\psi} w\left(u\left(c, z, G; \psi\right)\right) f(z, \psi)\, d\psi\, dz$$

If $w\left(u\left(c, z, G; \psi\right)\right) = u\left(c, z, G; \psi\right)$ the SWF is classical utilitarian

If $w''\left(u\left(c, z, G; \psi\right)\right) < 0$ the SWF is prioritarian

$f(z, \psi)$ is the joint density function

Optimal tax policy results

We obtain exactly the same optimal taxation rule as in a model without public goods, and the same as in, e.g., Saez (2001)

Not super-surprising

The optimal marginal taxation *rule* is in fact the same for all given public good provision levels, optimal or not

(Yet, the *levels* of the optimal marginal tax rates are of course affected by the public good provision)

A perturbation in the public good provision, *G*

Consider next a small increase, or perturbation, of the provided public good, *dG*

This will cause two welfare effects:

1. Increased welfare through individual's utility increase of the additional public good, expressed in units of public funds
2. A drop in the public budget

The latter is due both to its direct effect (the cost of providing the good) and indirect effects in terms of adjusted taxable incomes

Let the conditional mean of the *MWTP* for the public good at income z be given by

$$MRS_{G,c}(z,G) = \int_{\psi} MRS^{\psi}_{G,c}(z,G)\; s\left(\psi \middle| z\right) d\psi$$

Then we can formulate the optimal provision condition in terms of a single integral

Proposition 2. *The optimal provision of a public good for a given income tax schedule, optimal or not, is given by*

$$\int_0^\infty MRS_{G,c}\left(z,G\right)h(z)\,dz = 1 - F + D$$

where $F = \int_0^\infty T'\left(z\right)z_G(z,G)h(z)\,dz$, $D = \int_0^\infty \left(1-g\left(z\right)\right)MRS_{G,c}\left(z,G\right)h(z)\,dz$

where g is the derived welfare weight

The Samuelson rule is thus modified with two terms:

1. A fiscal externality term, F
2. A distribution adjustment term, D

This holds for any given tax system, optimal or not!

Next, let us assume income tax optimality

Define the within-individual *MWTP* income elasticity as

$$\alpha^{ind}(z) = \left(\frac{dMRS_{G,c}(z,G)}{dz} \right)^{ind} \frac{z}{MRS_{G,c}(z,G)}$$

and the corresponding between-income elasticity

$$\alpha^{cross}(z) = \frac{dMRS_{G,c}(z,G)}{dz} \frac{z}{MRS_{G,c}(z,G)}$$

Let $\varsigma^{c}(z)$ be the compensated taxable income elasticity

Proposition 3. *The optimal provision of a public good for an optimal income tax schedule is given by*

$$\int_0^{\infty} MRS_{G,c}\left(z,G\right)h(z)\,dz = 1$$

$$+\int_0^{\infty} \varsigma^{c}(z)\frac{T'\left(z\right)}{1-T'\left(z\right)} MRS_{G,c}\left(z,G\right)\left(\alpha^{cross}(z)-\alpha^{ind}(z)\right)h^{*}(z)\,dz$$

Thus, if $\alpha^{cross}(z) > \alpha^{ind}(z)$ for all income levels, the public good should be underprovided relative to the Samuelson rule, and vice versa

Separating Exposure from Preference Heterogeneity

Suppose now that preferences are identical and moreover weakly separable as assumed by Christiansen (1981)

$$u(f(c,\hat{G}),z;\psi)$$

but where exposure to the public good differ

$\hat{G}$ is the exposure-adjusted public good, such that $\hat{G}=\phi G$, where ϕ varies with income but is constant for all individuals with the same income

Let us define an elasticity reflecting how ϕ varies with income, as follows:

$$\varepsilon^{\phi}(z) = \frac{d\phi}{dz}\frac{z}{\phi}$$

Let us also define the income elasticity of the *MWTP* for the exposure-adjusted public good

$$\alpha^{\hat{G}}(z) = \frac{dMRS_{G,c}\left(z,\hat{G}\right)}{dz}\frac{z}{MRS_{G,c}\left(z,\hat{G}\right)}$$

Proposition 4. *The optimal provision of a public good for an optimal income tax schedule when individuals have identical and separable underlying preferences but income-dependent exposures to the public good is given by*

$$\int_0^\infty MRS_{G,c}\left(z,G\right)h(z)\,dz = 1+\int_0^\infty \varsigma^c(z)\frac{T'\left(z\right)}{1-T'\left(z\right)}MRS_{G,c}\left(z,G\right)\varepsilon^\phi(z)\left(1+\alpha^{\hat{G}}(z)\right)h^*(z)\,dz$$

The public good should then be overprovided relative to the Samuelson rule when low-income individuals have a higher exposure to the public good ($\varepsilon^\phi(z)<0$), and vice versa

Conclusion

Contrary to the conventional view, it is often optimal to take distributional concerns into account also in cost-benefit analysis, and thus not to delegate such concerns solely to the tax and transfer system

This is in particular the case when the exposure to the public good is strongly income-dependent

This holds also under optimal non-linear income taxes when people have identical and separable preferences!

Thanks for listening!