Tax Progressivity and Income Inequality: A Simple Formula

Julius J. Andersson

Stockholm School of Economics

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This Paper: A Simple Formula for Tax Progressivity

The Kakwani index of tax progressivity for indirect taxes can be approximated as:

$$K \equiv C(T) - G \simeq (\eta - 1)G$$

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Empirical test: Sweden's carbon tax on fuel and VAT on food

- Prediction: Regressing K on $G \rightarrow$ linear relationship and slope $(\eta - 1)$

Motivation: Why does Tax Progressivity Vary so Much?

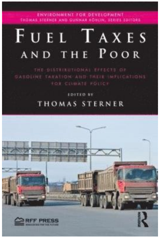
- A central question in public economics: the distributional effects of taxation
- Progressivity of **income taxes**: reflects **statutory tax schedule**
- Progressivity of **indirect taxes**: uniform tax rates across households $\rightarrow$ departures from proportionality arise from **behavioral differences**
- Behavioral foundation implies wide variation in tax progressivity across economic contexts. Example: **carbon and transport fuel taxes**

$\Rightarrow$ How can we explain this variation in tax progressivity?

Motivation: Why does Tax Progressivity Vary so Much?

The puzzle:

- The same tax (e.g., fuel tax) can be:
  - Regressive in the US
  - Proportional in the Nordics
  - Progressive in developing countries



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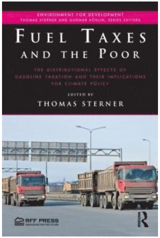

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   "a measure of tax progressivity should depend on the magnitude of the difference of the tax elasticity from unity" – (Kakwani, 1977)

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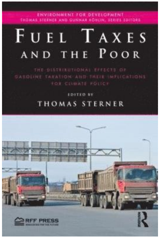

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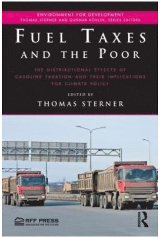

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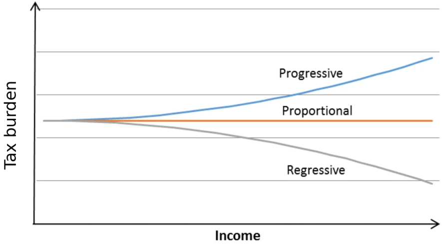
However: *"there is no very obvious relation"* between tax progressivity and inequality (Sterner et al., 2012).

This paper: Unifies tax progressivity, η, and G into a simple formula.

Measuring Tax Progressivity: the Kakwani Index

- If not proportional, taxes are either regressive or progressive



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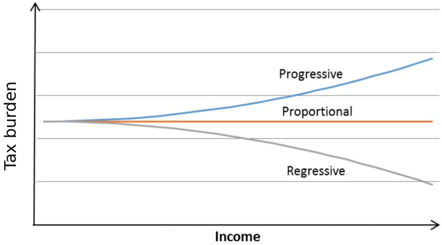

- One popular summary measure: the **Kakwani index** (Kakwani, 1977)

$$K = C(T) - G$$

$C(T)$: the concentration index of tax payments T

G: pre-tax Gini coefficient (measure of income inequality)

Kakwani: the gap between tax concentration curve and Lorenz curve

- $K > 0$ progressive; $K = 0$ proportional; $K < 0$ regressive

Deriving the Simple Formula: Two Ingredients

Goal: Derive $C(T) \approx \eta\, G$

Households have disposable income y, ranked from poorest to richest with fractional rank $R \in (0, 1]$, and with mean $\mathbb{E}[R] = 1/2$

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1. **Log-linear Engel curve** for pre-tax expenditure on the taxed good:

$$c(y) = A\, y^{\eta}, \qquad A > 0,\ \eta \in \mathbb{R} \tag{1}$$

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2. **Covariance forms** of concentration indices and Gini:

$$C(T) = \frac{2}{\mu_T}\ \mathrm{Cov}(T, R), \qquad G = \frac{2}{\mu_y}\ \mathrm{Cov}(y, R) \tag{2}$$

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Which gives Kakwani index in covariance form:

$$K = \frac{2}{\mu_T}\ \text{Cov}\big(T, R\big) - \frac{2}{\mu_y}\ \text{Cov}(y, R) \tag{3}$$

Deriving the Simple Formula: Tax Payments

Then, a proportional excise tax with rate τ is imposed on the good

- Tax payments:

$$T(y) = \tau\, c(y) = \kappa\, y^{\eta}, \qquad \kappa \equiv \tau A \tag{4}$$

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- And thus:

$$K = \frac{2}{\mathbb{E}[y^{\eta}]}\, \mathrm{Cov}\big(y^{\eta}, R\big) - \frac{2}{\mu_y}\, \mathrm{Cov}(y, R) \tag{6}$$

- Note: κ cancels out in equation (5) $\rightarrow$ the Kakwani index is invariant to the tax level (progressivity is about *relative* burden across ranks)

Deriving the Simple Formula: Linearization

- Linearizing y^η around mean income μ_y:

$$y^\eta \approx \mu_y^\eta + \eta\,\mu_y^{\eta-1}(y - \mu_y), \tag{7}$$

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- Substituting into (5) yields the key approximation:

$$C(T) \approx \eta\, G \tag{8}$$

- And hence, the Kakwani index simplifies to:

$$K = C(T) - G \approx (\eta - 1)G \tag{9}$$

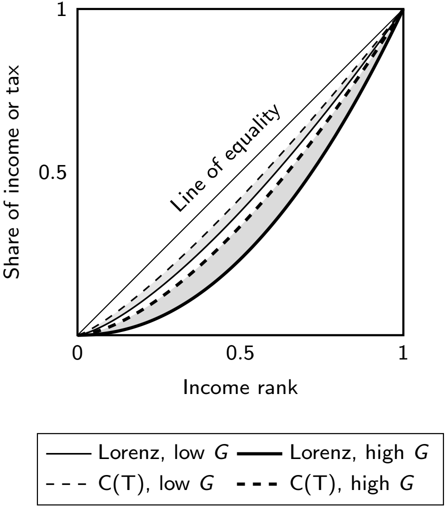
Interpretation: Elasticity gap and Inequality

$$K \simeq (\eta - 1)G$$

- $(\eta - 1)$: behavioral component (how spending shifts with income). Measures the **elasticity of the budget share** for the taxed good
- G: income inequality is a (distributional) amplifier
- Their product measures tax progressivity

Graphical Intuition: Lorenz vs. Tax Concentration Curves

$\eta < 1$: Necessity (Regressive)



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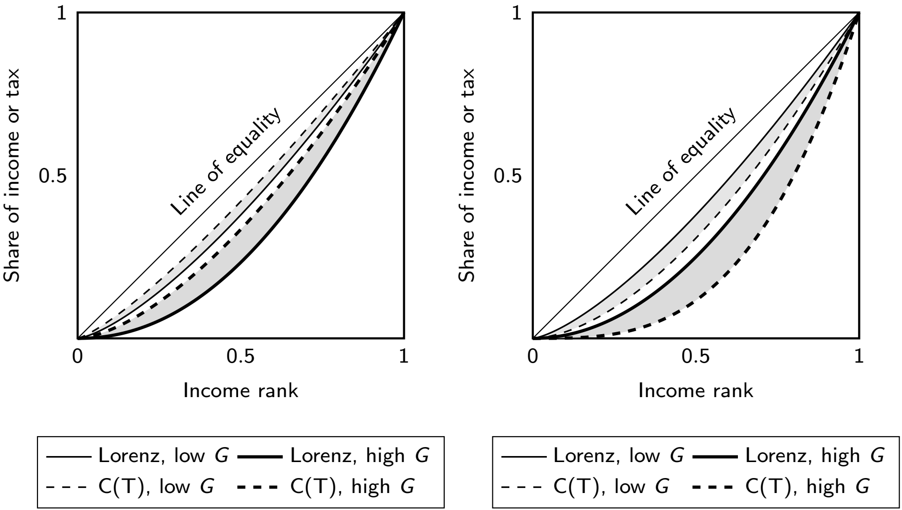

Extension: Heterogeneous Income Elasticities

- Preceding analysis assumed a **constant** income elasticity of demand
- Now, allow: $\eta = \eta(y)$ (**heterogeneous** income elasticities)
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- Then, it follows that:

$$K \simeq (\bar{\eta}_R - 1)G \tag{10}$$

- Where $\bar{\eta}_R$ is a **rank-weighted** average elasticity
  - Intuition: rising inequality puts more rank-weight on the rich; if $\eta(y)$ falls with income, $\eta'(y) < 0$, the effective elasticity declines

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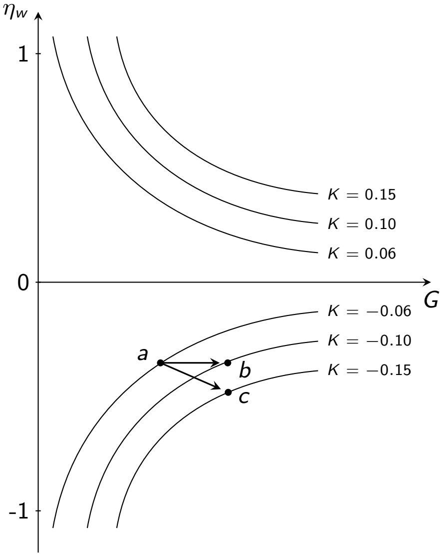
- Rising inequality affects tax progressivity through two channels:
    1. **Direct effect** through higher G
    2. **Compositional effect** by shift in $\bar{\eta}_R$

Isoprogressivity Curves: Direct and Compositional Effects



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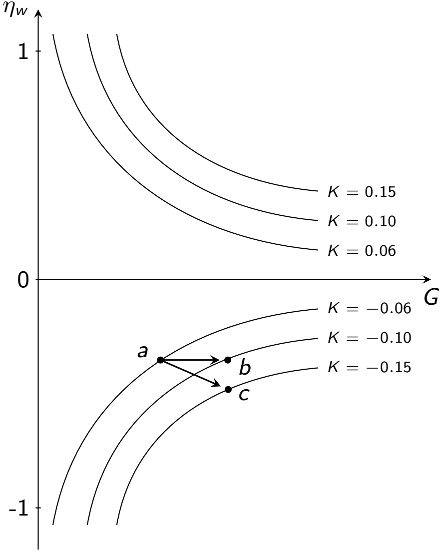

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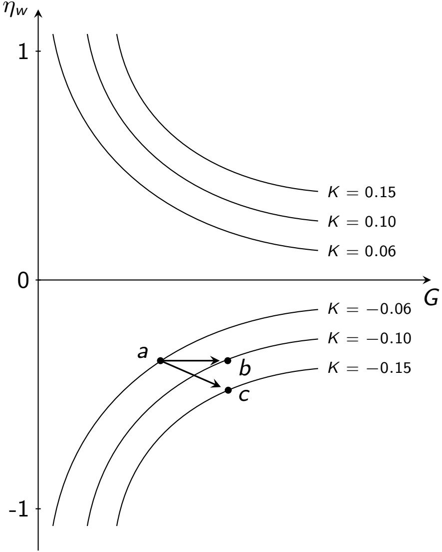

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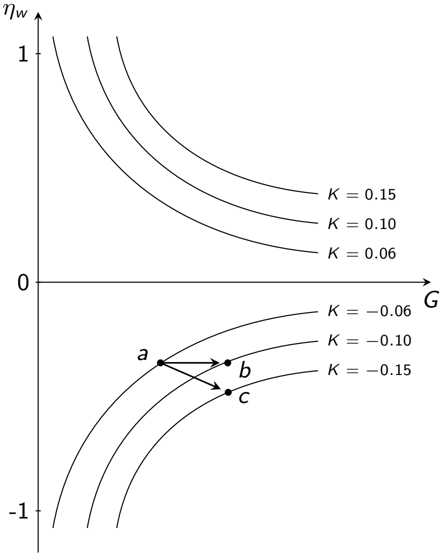

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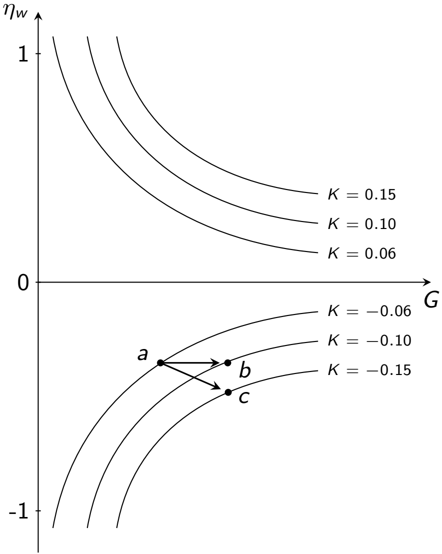

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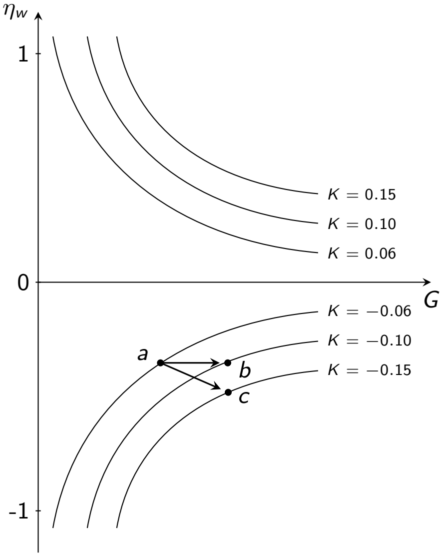

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- **Total effect (a→c)**: $\Delta K_{\text{total}} = -0.09$

Empirical test: Sweden as a Case Study

Empirical prediction:

- If η is **constant**:
  - K is linear in G, with slope $(\eta - 1)$ and intercept around zero
  - Slope maps to implied elasticity: $\eta = 1 + \textit{slope}$
- If elasticities are **heterogeneous** $\eta(y)$:
  - Still linear relationship but slope reflects the compounded effect ($\bar{\eta}_R$ moves with G)
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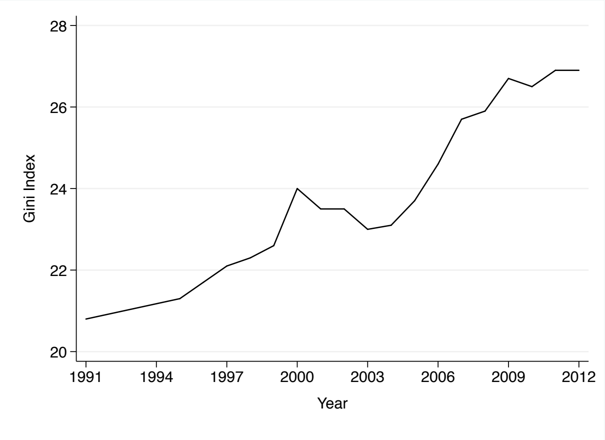
Case study: Sweden's carbon tax on transport fuel and VAT on food

- Carbon tax on transport fuel since 1991
- VAT of 12 percent on all food products
- Data: household survey data 1999-2012 for carbon tax, 2003-2012 for VAT on food (source: Statistics Sweden)

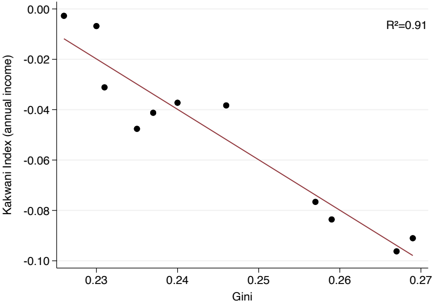
Empirical test: Income Inequality in Sweden



Gini coefficient in Sweden: 1991-2012

- Variation in Gini during sample years (1999-2012): 0.22-0.27
- Both increases and decreases in inequality

Empirical test: Carbon Tax on Fuel and VAT on Food



(a) Carbon tax on transport fuel

- Average $\eta = 0.80$ (necessity)
- Slope implies $\eta = -1$
- Indicates: heterogeneous $\eta(y)$

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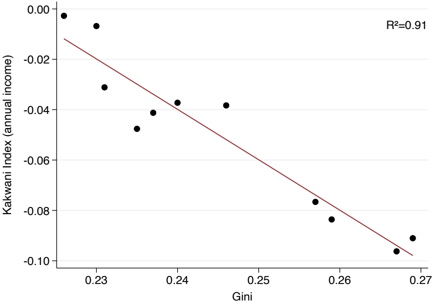

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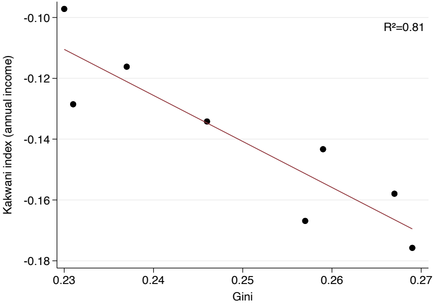

(b) VAT on food

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- Average $\eta = 0.44$ (necessity)
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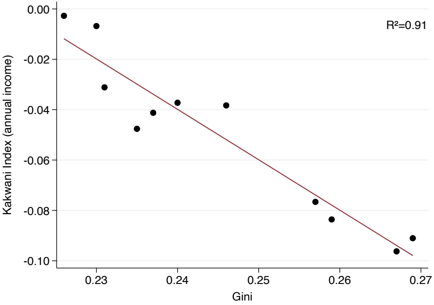
Empirical test: Carbon Tax on Fuel and VAT on Food



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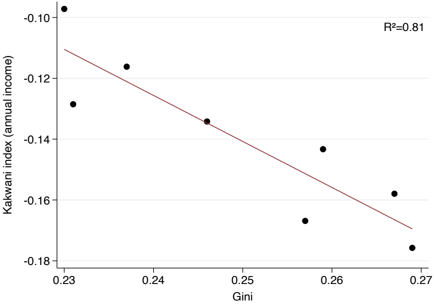

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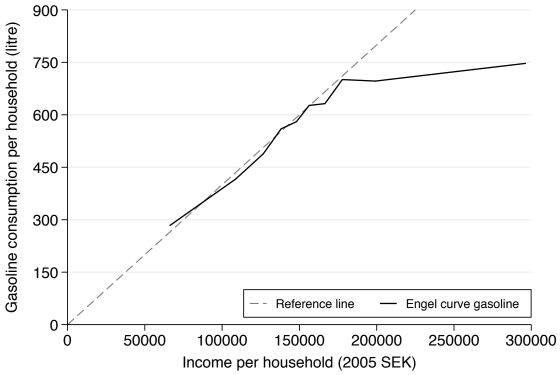
Finding average η? $\Rightarrow$ Regress K on G with "noconstant"

Gasoline Engel Curve and Evidence on $\eta(y)$



- Reference line (benchmark): $\eta = 1$
- Engel curve flattens at higher incomes: Consistent with $\eta(y)' < 0$

Contributions to Literature

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4. Simplifies the Reynolds-Smolensky index (full distributional effect):

$$RS = G_{\text{pre}} - G_{\text{post}} = \frac{g}{1-g} K \Rightarrow \boxed{RS \simeq \frac{g}{1-g} (\eta - 1)\, G}$$

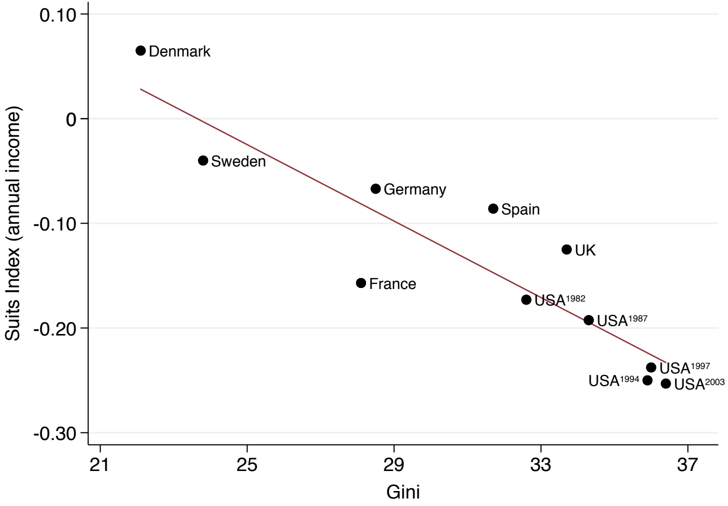
where g is the average tax rate.

Conclusion with Policy Implications

$$\boxed{K \simeq (\eta - 1)G}$$

1. Formalizes intuition: tax on necessities is regressive, tax on luxuries progressive
2. Inequality alone can shift tax progressivity of existing taxes
3. Explains cross-country variation: same tax $\rightarrow$ different progressivity under different η and G (e.g., regressive carbon tax in the US, proportional in the Nordics)
4. More speculative: Matters for the sustainability of climate policy. May explain cross-country variation in political acceptance of carbon and fuel taxes

Tax Progressivity and Inequality: Cross-Country Evidence



Note: tax progressivity of gasoline taxes measured using Suits index.

Deriving the Simple Formula: $C(T) = \eta G$

- Linearizing y^η around mean income μ_y:

$$y^\eta \approx \mu_y^\eta + \eta\,\mu_y^{\eta-1}(y - \mu_y), \tag{11}$$

- Which implies:

$$\text{Cov}(y^\eta, R) \approx \eta\,\mu_y^{\eta-1}\,\text{Cov}(y, R), \quad \text{and} \quad \mathbb{E}[y^\eta] \approx \mu_y^\eta \tag{12}$$

- Substituting into (5) yields the key approximation:

$$\begin{aligned} C(T) &= \frac{2}{\mathbb{E}[y^\eta]}\,\text{Cov}(y^\eta, R) \approx \frac{2}{\mu_y^\eta}\Big(\eta\,\mu_y^{\eta-1}\,\text{Cov}(y, R)\Big) \\ &= \eta\,\frac{2}{\mu_y}\,\text{Cov}(y, R) = \eta\,G, \end{aligned} \tag{13}$$

- And hence, the Kakwani index simplifies to:

$$K = C(T) - G \approx (\eta - 1)G \tag{14}$$

Limitations and Assumptions

The simple formula is a first-order approximation rather than an exact identity.

Assumptions:

- Moderate income dispersion and a locally log-linear Engel curve
- That η is constant, if not, η reflects a rank-weighted average elasticity
- Full tax pass-through to consumers
- Fixed (pre-tax) disposable income with no behavioral feedbacks
- A single taxed good
- No re-ranking