Intergenerational Discounting and Inequality

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Motivation

- Policies enacted today **shift resources between generations**.
- Examples:
  - Mitigation of climate change;
  - Public debt reduction;
  - Research and development for technological change;
  - Pension reform.

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- Evaluating such policies requires ethical judgments of **two distinct kinds**:
  - Trade off earlier vs. later generations—*discounting*;
  - Trade off better-off vs. worse-off generations—*inequality*.

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  - Trade off better-off vs. worse-off generations—*inequality*.
- The applied literature proceeds by selecting a small set of familiar criteria and conducting sensitivity analysis over a few parameters (Stern, 2007).
- This restricts attention to a **narrow subset of discounting and inequality attitudes**.

Focus of the paper

- Our contribution is to **make the underlying ethical choices explicit**.
- We **characterize the full class of criteria** in which discounting and inequality can be specified independently, compared transparently, and combined in tractable ways.

Our contribution

1. We propose **"modular" theories of intergenerational justice**:
   - 1.1 *Module 1* exclusively deals with **discounting**;
   - 1.2 *Module 2* exclusively deals with **inequality**;
   - 1.3 Any theory is fully described by these two modules;
   - 1.4 It reveals a wealth of new criteria.
2. We **axiomatically characterize** the class of modular theories of intergenerational justice (Pareto and "nested time split"); modularity uniquely identified.

Our contribution

3. We offer **meaningful comparisons** of social impatience and inequality aversion across criteria.
4. We derive an **equity-efficiency decomposition** and a **generalized Ramsey rule**.
5. We **extend the framework beyond Pareto**, which connects our approach to the classic literature (Koopmans, 1960; Diamond, 1965).

Disentanglement

A consumption stream

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- A theory of intergenerational justice is represented by a **social welfare function**

$$W : C \to \mathbb{R}.$$

We assume that the social welfare function is continuous.

Change of perspective

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  - Instead of comparing consumption over calendar time by their present value, we compare them by their duration in equivalent time.

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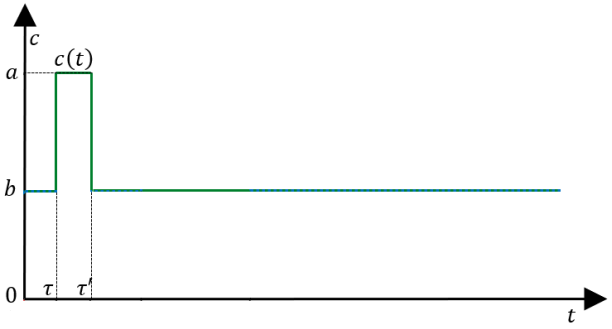

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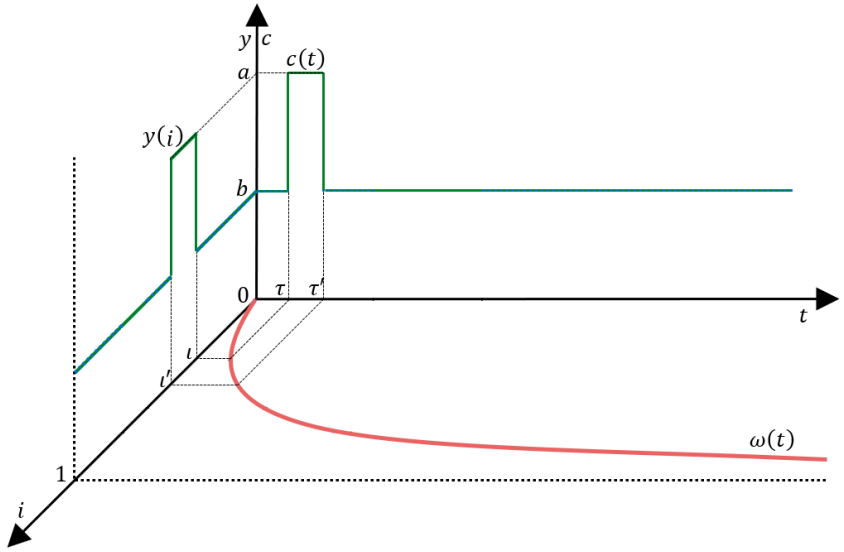

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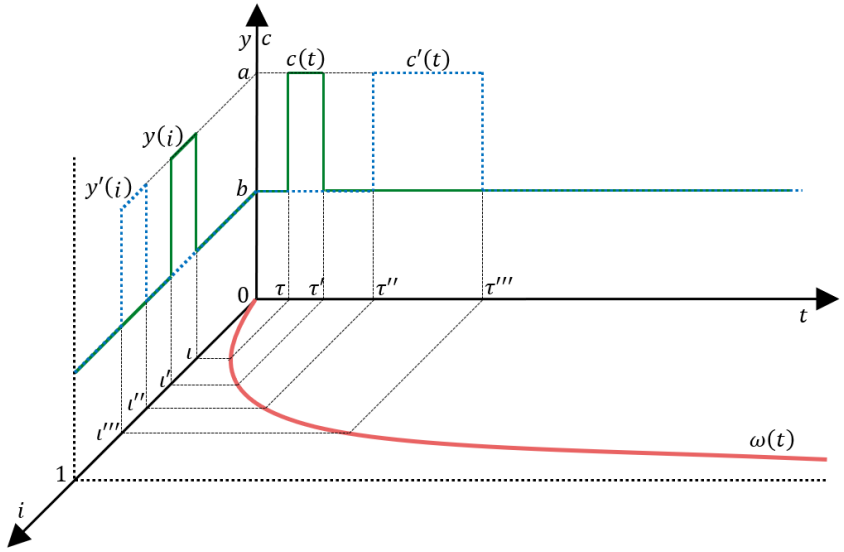

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  - Instead of comparing consumption over calendar time by their present value, we compare them by their duration in equivalent time.
- *Anticipating the results:* once consumption is reparametrized, inequality aversion can be expressed by a standard law-invariant aggregator.

The time-discounting function

- We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing, continuous, onto map $\omega : T \to I$, such that rearrangements on I is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y \circ \omega, c' = y' \circ \omega, y = \Pi y'.$$

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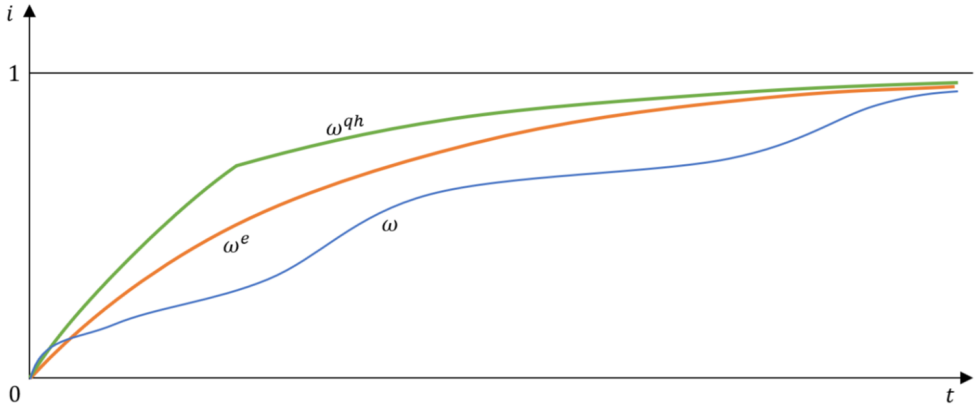
- We call ω the (cumulative) **time-discounting function**.

The time-discounting function



$\omega^e(t) = 1 - e^{-\rho t}$ $\omega^{qh}(t) = \begin{cases} \beta\kappa(1 - e^{-\rho t}), & \text{for } t \in [0, \tau] \\ 1 - \kappa e^{-\rho t}, & \text{for } t > \tau, \end{cases}$ with $\kappa = 1/(\beta + (1 - \beta)e^{-\rho t})$

The aggregator

- How to aggregate consumption streams in equivalent time?
- Given the social welfare function W and the time-discounting function ω, the **aggregator** $V : Y \to \mathbb{R}$ immediately follows by setting

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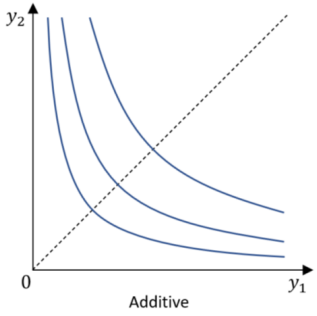
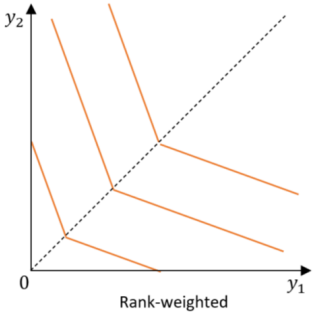
- Note that the aggregator is *law invariant* in equivalent time by definition.
- Thus, V could be any standard (atemporal) social welfare function.

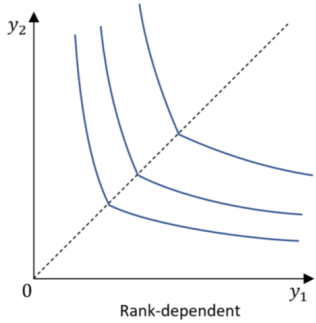
The aggregator



$$V^a = \int_0^1 \frac{(y_i)^{1-\eta}}{1-\eta} di$$



$$V^{rw} = (\gamma+1)\int_0^1 \big(1-r(i)\big)^{\gamma} y_i di$$



$$V^{rd} = (\gamma+1)\int_0^1 \big(1-r(i)\big)^{\gamma} \frac{(y_i)^{1-\eta}}{1-\eta} di$$

The modular social welfare function

- We say that W is **modular** if consumption streams can be represented in equivalent time and, thus, there exist an ω and V induced by W.
- Importantly, the most common welfare criteria adopted in the literature are modular.

Example formulas

Characterization

Nested time split

- We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing Pareto and the novel axiom of **nested time split**.

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- We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing Pareto and the novel axiom of **nested time split**.
- It requires that **any calendar time interval** can be divided into two sub-intervals of "equal value".
- A social welfare function W satisfies **nested time split** if for every nondegenerate $\mathcal{A} = [\underline{t}, \overline{t}) \subseteq T$ there exists $t^* \in (\underline{t}, \overline{t})$ with $T_1 = [\underline{t}, t^*)$, $T_2 = [t^*, \overline{t})$ such that, for each consumption stream $c \in C$ and each pair of consumption levels $\underline{c}, \overline{c} \in \mathbb{R}_+$,

$$W(\underline{c}_{T_1}, \overline{c}_{T_2}, c_{T/\mathcal{A}}) = W(\overline{c}_{T_2}, \underline{c}_{T_1}, c_{T/\mathcal{A}}).$$

Nested time split

- **Theorem 1.** Assume W is Paretian. W is modular iff W satisfies nested time split.
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- **Theorem 1.** Assume W is Paretian. W is modular iff W satisfies nested time split.
- **Corollary 1.** Assume W is Paretian. If W satisfies nested time split, then ω and V are unique.
- *Intuitively:* Pareto rules out intervals of calendar time that are ethically irrelevant.
- Nested time split then forces the relative weight of adjacent sub-intervals to be independent of the rest of the consumption stream.

Generalized modular: relaxing Pareto—slow, zero, or negative discounting

Dynamic axioms: stationarity, time consistency and time invariance

Implications

Some modular theories of intergenerational justice

W	V	Additive V^a	Rank-weighted V^{rw}	Rank-dependent V^{rd}
ω		$\int_0^1 f(y_i)di$	$\int_0^1 g(r(i))y_i di$	$\int_0^1 g(r(i))f(y_i)di$
Exponential ω^e	$1-e^{-\rho t}$	EDU		
Quasi-hyperbolic ω^{qh}	$\begin{cases}\beta k(1-e^{-\rho t}) & t \le \tau \\ 1-ke^{-\rho t} & t > \tau\end{cases}$			
General ω				

NEW

A tractable family: $W^{erd} = (1+\gamma)\int_0^\infty e^{-\rho t}\left(1-r\left(\omega^e(t)\right)\right)^\gamma \frac{(c_t)^{1-\eta}}{1-\eta}\,dt$

Efficiency–equity representation

- **Proposition 1.** Assume W is modular, Paretian and scale invariant. Then the efficiency–equity representation satisfies:

$$W(c) = m(y) \cdot E(y), \qquad y = c \circ \omega^{-1}.$$

- Here, m is the mean and E is the equity factor (or 1 minus inequality), building on Atkinson (1970).

Generalized Ramsey rule

- **Proposition 2.** Assume W is modular and Paretian, and ω, V have "right-derivatives". Then the social discount rate satisfies:

$$SDR_t = \frac{1}{t}\ln(\omega'_+(0)/\omega'_+(t)) + \frac{1}{t}\ln(\mu(0;y)/\mu(\omega(t);y)).$$

- The first term captures the "pure time preference", embodied by the time-discounting function.
- The second term captures "local inequality aversion" ($\mu(i;y)$ is the marginal of V at i given y).
- For parametric rank-dependent families: $SDR_t^{erd} = \rho + \eta g_t^c + \gamma g_t^r$.

Conclusion

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- We propose modular theories of intergenerational justice.
- Distinctive features:
  - It builds on and highlights a variety of attitudes towards discounting and inequality;
  - It is very general and flexible, and enriches the toolbox of economic analysis. At the same time, the parametric specifications of these theories remain suitable for applied work.
- Estimation, application, and generalization of the modular theories also open many avenues for future research.

Comparisons: impatience, inequality aversion