Fear and Economic Behavior

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Abstract: Fear is an important factor in economic decision making, and may for example influence investments, conflicts, crime, and politics. I model strategic interactions between players who can be either in a neutral or a fearful state of mind. The state of mind determines the players' utility functions. My two main assumptions are that a fearful player is more concerned with risk and that fear is triggered after a sufficiently large increase in the player's expected cost of negative outcomes. I normalize the payoffs such that only outcomes bad enough to, potentially, instill fear when anticipated have a negative payoff. A player's beliefs over the expected cost of negative outcomes determine the player's transitions between the states of mind, and I use psychological game theory in the analysis of my applications. I show how fear can spread among bank clients and cause a bank panic, and how a player can use fear to bring about a desired outcome. I also illustrate the tendency of fear to amplify the behavioral response to an adverse event.

Keywords: emotions; fear; risk aversion; psychological games

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1 Introduction

Fear influences one's judgement, behavior, and decision making. A fearful person shows an increased concern with risk and tends to be less willing to participate in risky lotteries. For instance, a person who becomes fearful after an extreme movement on the stock market may overreact and sell more than he or she otherwise would have done. Although fear is an important factor in decision making under risk and uncertainty, few economists have studied the topic and a formal analysis of the behavioral consequences of fear remains absent.[1]

In this paper I propose a game theoretic model of players who can transition between a neutral and a fearful state of mind. A player's state of mind determines his or her utility function. While fear can manifest itself in various ways, in this paper I define fear as an emotion with a specific stimuli and starting point. In addition, rather than focusing on the disutility of experiencing fear, my focus is on the behavioral consequences of fear as mediated by an increased concern with risk.[2] Psychology literature emphasize that once an individual becomes fearful, his or her concern with risk increases.[3] I assume players in the fearful state of mind experience highly intense fear such that their coefficient of risk aversion goes to infinity. In the limit the utility function coincides with the maximin utility function. In the neutral state of mind, the players are risk neutral and their utility is linear in their expected material payoff. This assumption of risk neutrality is not crucial for the results. What matters is that the degree of risk aversion differs between the state of mind.

A player transitions from the neutral to the fearful state of mind after a sufficiently large increase in peril. I define peril as the expected cost of negative outcomes. The payoffs are normalized such that only outcomes bad enough to potentially instill fear when anticipated have a negative material payoff. Each player holds an initial belief over the peril of the interaction when the game

[1]The interest for fear and decision making has grown among empirical and experimental economists during the decade since the financial crisis (see e.g. Callen et al., 2014; Campos-Vazquez & Cuilty, 2014; Cohn et al., 2015; Dijk, 2017; Guerrero et al., 2012; Guiso et al., 2018; Kuhnen & Knutson, 2011; Malmendier & Nagel, 2011; Nguyen & Noussair, 2014; Wang & Young, 2020). The relationship between fear and risk attitudes is not found in all papers (see e.g. Alempaki et al., 2019; Gärtner et al., 2017).

[2]I abstract from the disutility of experiencing fear to focus on the behavioral consequences once someone is fearful.

[3]See e.g. Holtgrave & Weber (1993); Lerner & Keltner (2000, 2001); Lerner et al. (2003); Loewenstein et al. (2001); Smith & Ellsworth (1985); Smith & Lazarus (1991).

begins. A player's transitions are determined by the increase in peril compared with the initial belief, and by the player's individual fear threshold. Since the players' beliefs directly affects their utility functions, the game is a psychological game (Battigalli & Dufwenberg, 2009; Battigalli et al., 2019).[4] Take for example a person who considers the peril of going for a walk in the park late at night. A negative outcome in this interaction is to be robbed of all of one's money. Peril may increase if one is approached by a stranger.

An alternative way to model this is to have players who always are in one of the two states of mind. A share of the players are always fearful and have a maximin utility function, and the remaining players are always neutral and maximizes expected material payoff. However, such a model would result in other predictions. First, in the robbery game, the neutral players would always go for a walk, whereas the fearful players would never go for a walk. By contrast, players who can transition to a fearful state of mind may go for a walk anticipating to transition to the fearful state if a robbery attempt occurs. These players are in the neutral state of mind when the game begins and maximizes expected material payoff by going for a walk since the probability of a robbery attempt is sufficiently small. The players transition to the fearful state of mind only if a robbery attempt occurs, and, once fearful, prefer to comply. Second, in the bank run game, players who are always fearful never deposit their money in the bank, and always withdraw. A player who is neutral when the game begins may deposit the money while anticipating to transition to the fearful state of mind if sufficiently many other players withdraw. Third, in the public health intervention, a fearful decision maker always takes the vaccine and is unaffected by the information. By contrast, a decision maker who transitions to the fearful state of mind after bad news may overreact.

I illustrate the role of fear in three applications. The first is a sequential interaction between a robber and a victim. This game illustrates how a player may use his or her knowledge of another player's fear sensitivity to bring about a desired outcome. The second application is a simplified version of Diamond & Dybvig's (1983) seminal bank run game with the addition of an exogenous risk that a player 'needs money tomorrow' and is forced to withdraw. This game illustrates how fear can affect the outcome also when players' incentives are

[4] For a general discussion of psychological game theory and its usefulness in modeling emotions and other belief-dependent motivations, see e.g. Battigalli & Dufwenberg (2020).

aligned. It also illustrates how bank runs become more likely when players are sensitive to fear. The third application illustrates the consequences of fear when a public health authority informs the population about a disease. This application highlights the tendency of fear to amplify a player's response to information that increases the player's expected cost of negative outcomes.

The observation that fear can be of importance in strategic interactions has been made before. Shelling (1960) discusses the strategic consequences of fear in a situation similar to the robbery game studied in this paper. Shelling considers a homeowner who investigates a noise at night with a gun in his hand, just to find a burglar, also armed with a gun. The situation has two equilibria. In one, no one shoots and the burglar leaves quietly. In the other, the homeowner and the burglar shoot each other. While non of them prefers the shooting equilibria, Shelling notes that they may shoot not by calculation, but by nervousness. This situation can be formalized using the model I propose in this paper. If either the homeowner or the burglar were to become fearful, then shooting becomes the unique equilibrium.

Psychological game theory has been used to model other emotions, for example guilt and anger (Battigalli & Dufwenberg, 2007; Battigalli et al., 2019). Naturally, anger is the one most closely related to fear. Battigalli et al. (2019) model players with a belief-dependent utility function that assigns a weight both to own and others' material payoff. In the absence of frustration, the weight on others' material payoff is zero. However, as a player's expected material payoff decreases, his or her frustration increases. As frustration increases, the player's negative concern for others' payoffs increases. Similar to the emotional players studied in this paper, the players in (Battigalli et al., 2019) form initial beliefs over how the interaction will play out, and a disadvantageous change alters their utility function. However, Battigalli et al. study players who become frustrated if their expected material payoff decreases, and I study players who transition to a fearful state of mind if their expected cost of negative outcomes increases. Moreover, while the triggers of frustration and fear are similar, frustration causes a player to have a negative concern for others' payoffs whereas fear causes an increased concerned with risk.

Caplin & Leahy (2001) propose a model of anticipatory emotions. They study a two-period model of lotteries with a mapping from physical lotteries to mental states. The decision maker's first-period utility decreases in the variance of the second-period realizations of the lottery. A decision maker who experience anxiety prior to the resolution of a high variance lottery has a lower first-period utility and is less likely to take part in lotteries with high variance. By contrast, in this paper I focus on the behavioral consequences once a player has transitioned to a fearful state of mind.

Another closely related paper is Kőszegi & Rabin (2007). Kőszegi & Rabin build on Kahneman & Tversky's (1979) work on prospect theory. They model a decision maker who evaluates an outcome relative to a reference point formed by the decision makers recent beliefs. The decision maker's utility is a combination of a reference-independent "consumption utility" and a "gain loss" utility that depends on the difference between the consumption utility and the reference point. The decison maker's reference point determines how much risk he or she is willing to take on. The reference point can be either deterministic or stochastic. A decision maker who expects risk views a lottery as less aversive than a decision maker who does not expect risk to start with. Moreover, the decision maker is sophisticated in the sense that he or she correctly predicts the environment and own behavior in the environment. Similarly, I model players whose transitions between the states of mind depend on the players' initial beliefs over how the interaction will play out. The players are also sophisticated in the sense that they correctly predicts own and others' state transitions and the behavioral consequences. However, while Kőszegi & Rabin's decision maker has a reference-dependent utility function, the players in my model have belief-dependent transition probabilities between emotional states of mind. Further, Kőszegi & Rabin's decision maker is concerned with expected consumption utility and any deviations therefrom, whereas the players in this paper are concerned with the expected cost of negative outcomes.

Dillenberger & Rozen (2015) also study decision makers whose risk attitude may change during the decision making process. They model a decision maker who makes repeated decisions over lotteries. The decision maker becomes more risk averse after a disappointing realization than after an elating. Realizations are classified as disappointing or elating using a threshold rule. By contrast, I study players who may transition to a fearful state of mind when the expected cost of negative outcomes increases, rather than by the realization of a negative outcome.

Related is also Chichilnisky (2009) who extends the notion of rationality with new axioms of choice that include sensitivity to rare and catastrophic events. The new axioms allow extreme responses to extreme events, and the decision criteria is a combination of expected utility and minimizing losses in the case of an extreme event. The extreme events can be interpreted as similar to the negative outcomes in this paper. However, while Chichilnisky extends the notion of rationality to acknowledge extreme events, I study players who react emotionally to an *increase* in their expected cost of negative outcomes. If the increase is sufficiently large, the player transitions to the fearful state of mind in which he or she has a maximin utility function.

This paper proceeds as follows. Section 2 presents the formalization of the causes and consequences of fear and the psychological games framework needed to model these. Sections 3, 4, and 5 apply the model to three situations, a robbery game, a bank run game, and a public health intervention, and analyze the play between fearful players. Section 6 discusses the model and concludes.

2 The Model

Game form The focus of this paper is on a class of finite multi-stage game forms with observed actions and perfect recall.[5] Let $I = 1, ..., n$ denote the finite set of personal players. The game form may contain chance moves or moves by "nature" denoted by player 0. Let $I_0 = I \cup \{0\}$.

The multi-stage game consists of $L+1$ stages indexed by $l \in \{0, ..., L\}$. At the end of each stage, all players observe the stage's action profile. Let $a^0 \equiv (a_0^0, a_1^0, ..., a_n^0)$ be the stage-0 action profile. At the beginning of stage 1, players know history h^1, which can be identified with a^0. Similarly, define h^{l+1}, the history at the end of stage l, to be the sequence of actions in the previous periods, $h^{l+1} = (a^0, a^1, ..., a^l)$. Let the initial, or empty, history be denoted by h^0, the set of non-terminal histories denoted by H, and let the set of terminal histories, or plays, h^{L+1}, be denoted by Z.

Let $A_i(h^l)$ denote the feasible actions of player $i \in I$ in stage l when the

[5] A game form (or a game protocol) specifies the structure of a strategic situation: the players, how they can choose, and the material consequences of their actions.

history is h^l, and let $A(h^l) = \times_{i \in I} A_i(h^l)$. In each stage the players, including chance, simultaneously choose actions from a finite subset of (potentially) history-dependent feasible actions $A_i(h^l)$. This can be done without loss of generality since an *inactive* player can be represented as a player whose feasible actions $A_i(h^l)$ is a singleton with "do nothing" as the only action.

If chance is active at h^l, it's move is specified by the probability mass function $p_0(\cdot|h) \in \Delta(A_0(h^l))$. Chance selects a feasible action at random, and the action is revealed to the players after the stage. The players have identical priors on the probability of chance's actions.

The *material* payoffs of the players' actions are determined by an outcome function $\pi : Z \to \mathbb{R}^n$ that associates each play z with a profile of material payoffs.

Beliefs Players form beliefs both over own and others' behavior, and over others' beliefs about behavior. A player's beliefs are modeled as a hierarchical conditional probability system (Battigalli et al., 2019). A player's beliefs over own and other's behavior – the plays $z \in Z$ – are called *first-order beliefs.* They are defined for each history $h \in H$. The first-order beliefs are denoted by $\alpha_i(\cdot|Z(h)) \in \Delta(Z(h))$, where $\Delta(Z(h))$ is the set of probability measures on $Z(h)$. The system of beliefs $\alpha_i = (\alpha_i(\cdot|Z(h)))_{h \in H}$ must satisfy two properties. First, Bayes' rule for conditional probabilities must hold whenever defined. Second, each player i must believe that other players' actions in the same stage are statistically independent of each other and of i's action.[6]

The first-order beliefs α_i, are composed of two parts: player i's beliefs over own and other's behavior. The beliefs over own behavior, $\alpha_{i,i} \in \times_{h \in H} \Delta(A_i(h))$, takes the form a behavior strategy. They can be interpreted as the player's *plan* since they are the result of the player's contingent planning of which action to take at each history.[7]

A player's beliefs over other players' first-order beliefs constitutes his or her *second-order beliefs.* Let $\Delta_{i,1}$ denote player i's space of first-order beliefs. Second-order beliefs are systems of conditional probabilities over both plays, $z \in Z$, and co-players' first-order beliefs, $\alpha_{-i} \in \times_{j \neq i} \Delta_{j,1}$, for each history $h \in H$. The

[6] See Battigalli et al. (2019) or Battigalli & Dufwenberg (2020) for a more detailed specification of the belief hierarchies and their properties.

[7] The beliefs over the behavior of others, $\alpha_{i,-i} \in \times_{h \in H} \Delta(A_{-i}(h))$, is also a behavior strategy if there is only one other player.

second-order beliefs are denoted by $\beta_i = (\beta_i(\cdot|h))_{h\in H} \in \times_{h\in H}\Delta(Z(h) \times_{j\neq i} \Delta_{j,1})$. Player i's space of second-order beliefs are denoted by $\Delta_{i,2}$. It can be shown that the first-order beliefs α_i can be derived from the second-order beliefs β_i such that beliefs of different orders are mutually consistent (see e.g. Battigalli et al., 2019).

Triggers of Fear Fear, according to psychology research, have a specific stimuli and a clear starting point. When the game begins, the players are in a neutral state of mind. The players can transition to a fearful state of mind during the game. The ultimate cause of the fear system is to detect warning signals for impending threats (see e.g. LeDoux, 1998). A wide variety of external stimuli may trigger a fear response.[8] Because fear stimuli differ between cultures and individuals, I model them as exogenously defined for each player and game, and treat them as primitives. The payoffs of each interaction are normalized such that only outcomes bad enough to, potentially, trigger a fear response when anticipated by player i have a negative material payoff for i.[9]

Player i's set of negative outcomes is defined as

$$Z_{i,-} := \{z \in Z : \pi_i(z) < 0\}. \tag{1}$$

Each player continuously assesses his or her situation to detect warning signals. I define player i's peril at history h, given his or her first-order belief system α_i as:

$$P_i(h|\alpha_i) = \sum_{z\in Z_{i,-}} \alpha_i(z|h)|\pi_i(z)|, \tag{2}$$

where $\alpha_i(z|h)$ is player i's belief, conditional on history h, in the negative outcome $z \in Z_{i,-}$, and $|\pi_i(z)|$ is the cost of the negative outcome. In other words, peril is the expected cost of negative outcomes. Peril increases in the probability of negative outcomes and in the cost of negative outcomes.A warning signal for player i at history h, given his or her first-order belief system α_i is defined as an increase in peril compared with the initial peril, $P_i(h^0|\alpha_i)$:

$$P_i'(h|\alpha_i) = \max\{0, P_i(h|\alpha_i) - P_i(h^0|\alpha_i)\}. \tag{3}$$

[8]Some common fear stimuli are snakes, heights, auto accidents, being in a fight, and losing one's job (Geer, 1965).

[9]A consequence of this modelling choice is increased modeller freedom. As will be shown, the set of equilibria is sensitive to small changes in payoff around zero.

The increase in peril required to cause a player to transition to the fearful state of mind may differ between players. Some people may be more sensitive to fear than others because of personality traits they are born with. I model this using a sensitivity parameter τ_i, which is interpreted as player i's fear threshold.[10] Player i transitions from the neutral to the fearful state of mind if $P_i'(h|\alpha_i) \geq \tau_i$. In other words, player i transitions when the increase in peril is more than he or she can bare. Moreover, the players are sophisticated in the sense that they correctly anticipates own and others' transitions between the emotional states of mind.

In other words, in this model, a player transitions to the fearful state of mind after a sufficiently large increase in probability or cost of negative outcomes. Consider again the player estimating the peril of taking a walk late at night. The negative outcome is to be robbed of all of one's money. In this model, if the player is sufficiently sensitive to fear, then fear is triggered when a stranger approaches, or, once approached, the sight of a knife. In addition, the more likely the player deems a robbery attempt a priori, the lower the increase in peril when the stranger approaches.Likewise, the more armed the player believes the robber to be, the lower the increase in peril by the sight of the knife. In particular, a player who is certain that he or she will be approached by a robber does not experience an increase in peril. The robbery attempt is already discounted.[11]

I assume that the interactions are fast enough for the initial peril to be the relevant reference point. Moreover, I assume that the player cannot transition from the fearful to the neutral state of mind. Since interactions are fast, there is not enough time for a player to calm down even if peril decreases. This is in line with research showing that behavioral consequences of emotions tend to linger after the source of the emotion has vanished (Andrade & Ariely, 2019).

Fear and Risk Preferences at a given node depend on expected material payoffs and the player's state of mind. I assume the players can transition between two states of mind: a neutral and a fearful. A player in the neutral state of mind

[10]A player's fear threshold, or fear sensitivity, can be thought of as related to the Big-Five personality trait neuroticism (Kumari et al., 2007).

[11]This is unintuitive. However, neuroscience research has shown that the amygdala's primary focus is on uncertainty. A subject who is uncertain over a dangerous outcome may experience fear while a subject who is certain over the dangerous outcome does not. An implicit assumption is that the players in this paper can both be certain that an outcome will occur and over what that outcome entails. That is rarely the case in real life.

maximizes own expected material payoff. A player in the fearful state of mind has an increased concern with risk. Psychology literature has long emphasized the relationship between fear and risk attitudes. Modern psychology literature uses the appraisal-tendency framework (Lerner & Keltner, 2000) to study emotions. This framework states that each emotion gives rise to a cognitive predisposition to appraise future events in line with one or more appraisal themes.[12] There are five central appraisal themes: certainty, pleasantness, attentional activity, anticipated action, and control. Fear is associated with a sense of uncertainty and a lack of a sense of control, both of which are factors that influence judgments of risk.[13]

The appraisal-tendency framework predicts that fear causes a person to be less willing to take risks. The effect of fear on risk attitude has been found to have two main mechanisms (see e.g. Cohn et al., 2015; Guiso et al., 2018; Lerner & Keltner, 2000, 2001; Lerner et al., 2003; Wang & Young, 2020). First, a fearful person tends to behave as if their risk aversion has increased. Second, the subjective probabilities the fearful individual assigns to negative outcomes increases. The focus of this paper is situations of risk rather than uncertainty and I model the effect of fear on behavior as increased risk aversion.

I assume that the players are either in the neutral or the fearful state of mind. In the neutral state of mind, the player is a 'standard' risk neutral player who maximizes own material payoff. In the fearful state of mind, the player experiences highly intense fear such that his or her coefficient of risk aversion goes to infinity, and the player views a gamble in terms of it's worst-case scenarios. A fearful player's utility function corresponds to the maximin utility function.[14]

Using this formalization of the causes and consequences of fear, a player's "decision utility" function can be defined. A player i moving at history h maximizes a belief-dependent decision utility function $u_i : A_i(h) \times \Delta_{i,2} \to \mathbb{R}$ for $i \in I$,

[12] Those familiar with Frijda's (1986) action-tendency framework for emotions might notice that the appraisal-tendency framework can be viewed as an extension of this earlier framework.

[13] Fear is often associated with unpleasantness, but sometimes also with pleasantness. I restrict the focus of this paper to the change in the utility function rather than the utility from experiencing fear.

[14] A fearful victim does not exclude entirely from consideration any action of an opponent, even if the equilibrium under consideration prescribes a zero probability to that action.

$h \in H$, defined by:

$$u_i(h, a_i; \beta_i) = \begin{cases} \min_{a_{-i} \in A_{-i}(h)} E[\pi_i | (h, a_i, a_{-i})] & \text{if } P_i'(h; \alpha_i) \geq \tau_i \\ \mathbb{E}[\pi_i | (h, a_i); \beta_i] & \text{otherwise ,} \end{cases} \tag{4}$$

where α_i is derived from β_i; $P_i'(h; \alpha_i)$ is the increase in peril at history h given beliefs α_i; and τ_i is i's fear threshold.

While fear is solely determined by the player's first-order beliefs, a player intending to use others' fear thresholds to own advantage forms second-order beliefs over the co-players' first-order beliefs. In games of complete information, such as the applications in this paper, the players' fear thresholds are common knowledge. Note that while each decision utility function is belief-independent, the transition between the states of mind is belief-dependent.

Because the players begin in the neutral state of mind and fear is triggered by an increase in peril, the decision utility at the root coincides with expected material payoffs. Further, fear is possible at end nodes, but cannot influence subsequent choices as the game is over. One might allow the anticipation of fear to be felt at end nodes to influence earlier decisions. However, my assumptions rule this out.

Solution Concept Because the transition between the states of mind is belief-dependent, the games are psychological games in the sense of Battigalli & Dufwenberg (2009). The solution concept I use is the sequential equilibrium (SE) for psychological games (Battigalli & Dufwenberg, 2009; Battigalli et al., 2019). The SE for psychological games is an extension of Kreps and Wilson's (1982) classical notion of a sequential equilibrium. The games analyzed are one-shot interactions, and the equilibria are interpreted as the commonly understood ways to play the game by rational agents.

I consider a complete information framework where the rules of the game and the players' (psychological) preferences are common knowledge.[15] An SE is an assessment: a profile of behavior strategies σ_i – the player's plans – and conditional second-order beliefs β_i such that σ_i is the plan $\alpha_{i,i}$ derived from the

[15]This is not a limitation to the model since it can be extended to games of incomplete information. For an analysis of a psychological game of incomplete information see Attanasi et al. (2016).

second-order belief β_i. While the SE concept gives equilibrium conditions for infinite belief hierarchies, the applications of this paper only depend on first- and second-order beliefs and the SE is defined up to second-order beliefs.

Definition 1 (Battigalli, Dufwenberg & Smith, 2019). Assessment $(\sigma_i, \beta_i)_{i \in I}$ is *consistent* if for all $i \in I$, $h \in H$, $a = (a_j)_{j \in I} \in A(h)$
1. $\alpha_i(a|h) = \Pi_{j \in I} \sigma_j(a_j|h)$,
2. $marg_{\Delta_{-i,1}} \beta_i(\cdot|h) = \delta\alpha_{-i}$;

here α_i is derived from β_i and $\delta\alpha_{-i}$ is the Dirac measure assigning probability 1 to $\{\alpha_{-i}\} \subseteq \Delta^1_{-i}$.

The first condition requires players' beliefs about actions to satisfy independence across co-players, and after a deviation of player j player i expects j to behave in the continuation game as specified by j's plan $\alpha_{j,j}$. Thus, all players have the same first-order beliefs.

The second condition requires players' beliefs about co-players' plans to be correct and never change, on or off the path. Any two players thus share the same initial first-order beliefs about any other player and every player is able to infer the correct beliefs of his or her co-players.

Definition 2 (Battigalli, Dufwenberg & Smith, 2019). Assessment (σ_i, β_i) is a *sequential equilibrium* if it is consistent and satisfies the following sequential rationality condition:
for all $h \in H$ and $i \in I(h)$, $\sup \sigma_i(\cdot|h) \subseteq \underset{a_i \in A_i(h)}{\arg\max} u_i(h, a_i|\beta_i)$.

This definition coincides with the traditional definition of sequential rationality when players have standard preferences. An SE always exists when the utility functions are continuous (Battigalli et al., 2019). The utility function analyzed in this paper is discontinuous around τ_i, the fear threshold. A consequence is that a SE does not always exist. The situation is illustrated in the first application. However, if τ_i is sufficiently large, such that it is greater than the maximum increase in peril and the players cannot transition to the fearful state of mind, then there always is an SE that coincides with the SE in the game between players with standard preferences.

3 Robbery

A walk in the park Consider the example of the person going for a walk in the park late at night, running the risk of being robbed. This interaction can be modeled as the game in Figure 1. There are two players, the robber (player 1) and the victim (player 2). The game has three stages. Player 1 is active in stage 0 and stage 2 (player 2's only action in these stages is "do nothing"). Only player 2 is active in stage 1 (player 1's only action is "do nothing"). The game has no chance moves and any risk is endogenous. Player 2's fear threshold is denoted by τ_2.

Player 1 first decides whether to attempt a robbery (a) or not (n). Player 2 decides, conditional on an attempt, whether to comply (c) or resist (r). Player 1 decides, conditional on player 2 resisting, whether to flee the scene (f) or use violence (v) to force the robbery.

The payoffs are normalized such that all outcomes following a robbery attempt have a negative payoff for player 2. If player 1 does not attempt a robbery, both players receive a zero payoff. If player 1 attempts a robbery and player 2 complies, then player 1 receives a payoff of 50, and player 2 a payoff of -50. If player 2 resists and player 1 flees, each receives a payoff of -10. Both players are better off without an attempt; player 1 avoids an unpleasant experience, and player 2 is not chased by the police. If player 2 resists and player 1 uses violence, then player 1 receives a payoff of -200 and player 2 a payoff of -500; player 2 is injured and player 1 is chased more fiercely by the police.

Clearly, the game between players with standard preferences has a unique SE in $((n, f), r)$. However, if player 2 is fearful at his or her decision node, then he or she maximizes the minimum payoff by choosing c.[16] Player 1 has an incentive to ensure that player 2 transitions to the fearful state of mind if his or her decision node is reached. As we will see, player 1 can do so by randomizing between n and a.

Player 2's first-order beliefs α_2 can be split into beliefs over own plan, $\alpha_{2,2}$,

[16] Remember that a fearful victim does not exclude entirely from consideration any action of an opponent, even if the equilibrium under consideration prescribes a zero probability to that action.

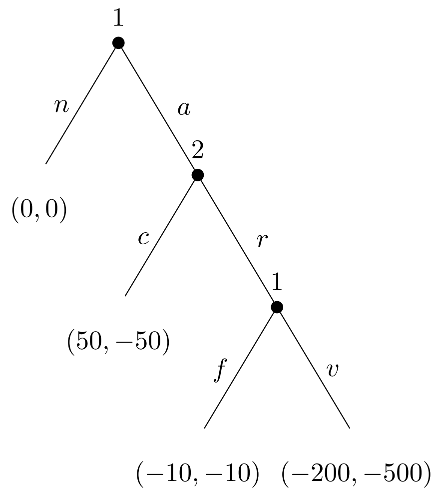

Figure 1: Robbery game.

and over player 1's plan, $\alpha_{2,1}$. Player 2's initial peril is

$$P_2\left(h^0|\alpha_2\right) = \left[50\left(1-\alpha_{2,2}^r\right) + \left(500\left(1-\alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r\right]\alpha_{2,1}^a, \qquad (5)$$

where $\alpha_{2,1}^a$ is player 2's belief that player 1 plans a, $\alpha_{2,2}^r$ is player 2's plan of choosing r; and $\alpha_{2,1}^f$ is player 2's belief that player 1 plans f, conditional on r.[17]

Player 2's initial peril is zero if player 2 believes that player 1 plans n. All negative outcomes follows a, and the expected cost of negative outcomes depend on player 2's beliefs over the three terminal histories (a, c), $((a, f), r)$, and $((a, v), r)$. Initial peril is strictly increasing in player 2's belief in a, $\alpha_{2,1}^a$. Moreover, peril depends on player 2's own plan of choosing c or r conditional on an a.

Player 2's updated peril if his or her decision node is reached is

$$P_2\left(a|\alpha_2\right) = 50\left(1-\alpha_{2,2}^r\right) + \left(500\left(1-\alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r. \qquad (6)$$

Since all negative outcomes follows a and all outcomes following a are negative, player 2's updated peril coincides with his or her expected material payoff. Player

[17]This is a slight abuse of notation, but the abbreviations used to denote the conditional probabilities of actions derived from players' plans are justified by the increased readability of the analysis.

2's increase in peril is

$$P_2'(a|\alpha_2) = \left(1-\alpha_{2,1}^a\right) \times \\ \left[50\left(1-\alpha_{2,2}^r\right) + \left(500\left(1-\alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r\right]. \tag{7}$$

The first factor is player 2's belief that player 1 will not choose a. The less probable player 2 believes a is, the larger the increase in peril if a occurs. If player 2 is certain that a will occur, $\alpha_{2,1}^a = 1$, then there is no increase in peril. The occurrence of a has already been taken into account. If player 2 is almost sure that a will not occur, $\alpha_{2,1}^a \approx 0$, then peril increases by close to the expected cost of the negative outcomes that follows a.

At the root of the game, Player 2 forms beliefs over player 1's plan. Player 1 maximizes the increase in peril, conditional on player 2's decision node being reached, by minimizing the probability of choosing a. Player 1's optimal plan is to maximize the probability of a, conditional on it being sufficiently small to ensure that player 2 transitions to the fearful state of mind, and to choose n only there is no way to ensure that player 2 transitions to the fearful state of mind. If player 1's second decision node is reached, then he or she maximizes own expected material payoff by choosing f.

Player 2's optimal plan depends on whether he or she transitions to the fearful state of mind after a. If player 2 is in the fearful state of mind, then he or she prefers c, otherwise he or she prefers r. If τ_2 is sufficiently large, such that it is above player 2's maximum increase in peril after a, then player 2 cannot transition to the fearful state of mind regardless of own and player 1's plans. If τ_2 is sufficiently small, then player 1 can randomize between n and a such that player 2 transitions to the fearful state regardless of own plan. However, for intermediate values of τ_2, whether or not player 2 transitions to the fearful state of mind depends on his or her own plan.

Equilibria When $\tau_2 > 50$, the game has a unique SE in $((n,f),r)$, just as in a standard game. To check this, note that when $\tau_2 > 50$, then player 2 remains in the neutral state of mind regardless of own and player 1's plan. If his or her decision node is reached, then player 2 maximizes own material payoff by choosing r. Player 1 knows this and maximizes own expected payoff by choosing (n,f). Further, $((n,f),r)$ remains an SE also when $10 < \tau_2 \leq 50$. For this intermediate range of τ_2, player 2's state of mind depends on his or her own plan. If player 2 plans r, then he or she transitions to the fearful state if $P_2'(a|\alpha_2) = 10(1-\alpha_{2,1}^a) \geq 10$, which only occurs if $\alpha_{2,1}^a = 0$. Hence, if player 2 plans r, then he or she remains in the neutral state after a and maximizes own material payoff by choosing r. Player 1 maximizes own expected payoff by choosing (n, f). In addition, when $10 < \tau_2 \leq 50$, then $\left(\left(\left[\frac{\tau_2}{50}, 1-\frac{\tau_2}{50}\right], f\right), c\right)$ qualifies as another SE. If player 2 plans c, then he or she transitions to the fearful state of mind after a, conditional on $P_2'(a|\alpha_2) = 50(1-\alpha_{2,1}^a) \geq 10$. Player 1 can randomize between n and a such that player 2 transitions to the fearful state. Player 1 maximizes own expected payoff by choosing $\left(\left[1-\frac{\tau_2}{50}, \frac{\tau_2}{50}\right], f\right)$. Player 2 transitions to the fearful state after a, and maximizes own minimum payoff by choosing c. Finally, when $\tau_2 \leq 10$, then $((n, f), r)$ cannot be an SE. To verify, assume it were. Player 2 plans r, and transitions to the fearful state if $P_2'(a|\alpha_2) = 10(1-\alpha_{2,1}^a) \geq \tau_2$. Player 1 can randomize between n and a such that player 1 transitions to the fearful state by choosing $\alpha_{1,1}^a \leq 1-\tau_2/10$. Player 2 transitions to the fearful state after a, and chooses c to maximize own minimum payoff, contradicting that $((n, f)r)$ is an SE. When $\tau_2 \leq 10$, then $\left(\left(\left[\frac{\tau_2}{50}, 1-\frac{\tau_2}{50}\right], f\right), c\right)$ is the unique SE. If player 2 plans c, then he or she transitions to the fearful state if $P_2'(a|\alpha_2) = 50(1-\alpha_{2,1}^a)$. Player 1 randomizes such that player 2 transitions and maximizes own minimum payoff by choosing c. □

In other words, whereas the game between players with standard preferences has a unique SE, the SE of the game between players who can transition to a fearful state of mind depends on player 2's fear threshold. If player 2 cannot transition to the fearful state of mind regardless of own plan, then the unique SE is identical to the SE in the game between players with standard preferences. For intermediate fear thresholds, the game has two SE. If player 2 believes that he or she will not transition to the fearful state after a and plans r, then he or she will not transition and prefers r. If player 2 believes that he or she will transition to the fearful state and plans c, then he or she will transition and prefers c, conditional on player 1's randomization. Due to the own-plan dependency of peril, player 2's beliefs are self-fulfilling.[18] Finally, if player 2 is sufficiently sensitive to fear,

[18]While games of complete and perfect information with no relevant ties always have a unique SE in standard games; this multiplicity of SE is not uncommon in psychological games and is due to own-plan dependency of the utility function.

then he or she may transition to the fearful state of mind, regardless of own plan. Player 1 randomizes such that player 2 transitions and the game has a unique SE in which player 1 chooses a with positive probability and player 2 chooses c.

Proposition 1. If τ_2 is sufficiently small such that he or she may transition to the fearful state of mind after an attempt, then an SE exists in which $\alpha^a_{1,1} > 0$ and $\alpha^r_{2,2} = 0$. If τ_2 is sufficiently small such that he or she can transition to the fearful state of mind after an attempt regardless of own plan, then this SE is unique.

Victims have time-inconsistent preferences. The victim is in the neutral state of mind when the game begins. The players are sophisticated in the sense that they correctly anticipates own (and others') future state of mind. Since the robber flees the scene if the victim resists, a victim in the neutral state of mind would prefer to commit to resisting a robbery attempt, fearful or not. If the victim were able to communicate such a commitment to the robber, the robber would prefer not to make an attempt. However, such a commitment is not possible in the scenario studied in this paper, and, once fearful, the victim complies with the robbery attempt.

Moreover, fear insensitive victims thus resists robbery attempts whereas victims more sensitive to fear complies. A fearful victim's compliance incentivizes robbers to seek out fear sensitive victims. The assumption that τ_i is common knowledge is crucial for this analysis. The caveat is that in reality, a robber finds it difficult to distinguish between fear sensitive and insensitive victim.

This interaction can be extended to a situation where player 1 is uncertain over player 2's fear threshold. Assume there is a population of potential player 2's and that their fear thresholds are uniformly distributed, $\tau_2 \sim U(0, 60)$. In other words, 1/6 are highly fear sensitive such that player 1 can always randomize such that they transition to the fearful state of mind should their decision node be reached. Another 1/6 are highly fear insensitive such that they cannot transition to the fearful state of mind. The remaining 4/6 may transition depending on their own plan. Assume, for simplicity, that half of them plans c and the other half r. Player 1 randomly selects a player 2 to approach (or not). Assume player 1, while not knowing the specific τ_2 of this player 2, knows the distribution of τ_2 in the population.

Player 1 increases the probability of player 2 transitioning to the fearful state of mind by decreasing the probability of choosing a. On the other hand, by increasing the probability of choosing a, more robbery attempts occur, and, as a consequence, more attempts are both resisted and complied with.

Player 1 chooses the probability of a that maximizes his or her expected material payoff

$$\max_{\alpha_{1,1}^{a} \in [0,1]} \left[50(\frac{6 - 5\alpha_{1,1}^{a}}{12}) - 10(1 - \frac{6 - 5\alpha_{1,1}^{a}}{12}) \right] \alpha_{1,1}^{a}. \tag{8}$$

The first term is the share of the population of player 2's that will transition to the fearful state of mind and choose c, given their (correct) beliefs $\alpha_{2,1}^{a}$ of player 1 choosing a. In other words, it is the share of the population with a fear threshold smaller than $50(1 - \alpha_{2,1}^{a})$, minus half of the population with $\tau_2 \in (10, 50]$ whose self-fulfilling beliefs lead them to choosing r over c. The second term is the share of the population of player 2's who remain in the neutral state of mind and chooses r. Conditional on r, player 1 chooses f, and receives a payoff of -10.

Player 1 maximizes his or her expected payoff by choosing $\alpha_{1,1}^{a} = \frac{2}{5}$. Player 2's with $\tau_2 \leq 10$ has a unique optimal plan in choosing c. Player 2's with $\tau_2 \in (10, 30]$ have self-fulfilling beliefs and transition to the fearful state of mind if they believe that they will transition and therefore plan to choose c. Consequently, the probability that player 1's attempt is successful is $1/3$.

Staying at home The game discussed above illustrates the intuition of the model, but it does not capture the full story of the person going for a walk late at night. More realistically, player 2 makes an initial decision of whether to go for a walk (g) or stay at home (s). The corresponding game is illustrated in Figure 2. Note that player 2 now makes the first decision. The game has four stages. Player 2 is active in stage 0 and 2, and player 1 in stage 1 and 3. Player 2 receives a material payoff of 5 from choosing g, conditional on n. If player 2 chooses s, then both players receive a zero payoff zero. Remaining payoffs are as in the previous example.

The game between players with standard preferences has a unique SE in $((g, r), (n, f))$. As before, the SE of the game between players who can transition to the fearful state of mind depends on τ_2.

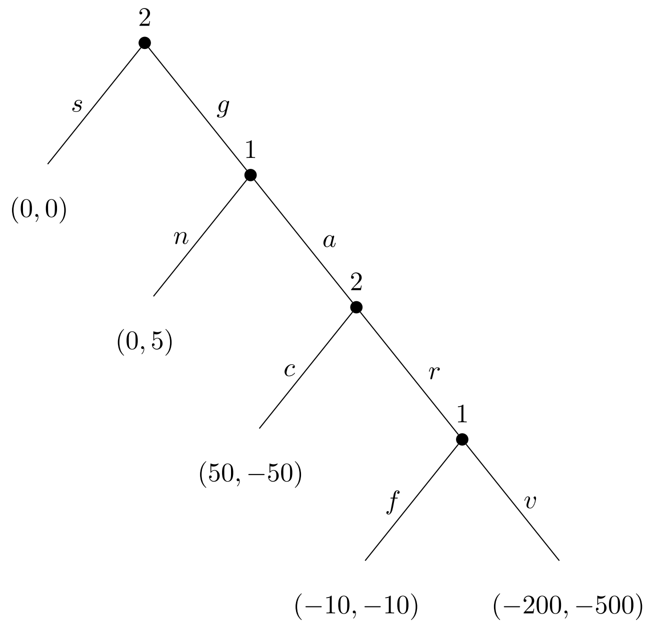

Figure 2: Extended robbery game.

Player 2's initial peril is

$$P_2\left(h^0|\alpha_2\right) = \left[50\left(1-\alpha^r_{2,2}\right) + \left(500\left(1-\alpha^f_{2,1}\right) + 10\alpha^f_{2,1}\right)\alpha^r_{2,2}\right]\alpha^g_{2,2}\alpha^a_{2,1}, \tag{9}$$

where $\alpha^g_{2,2}$ denotes player 2's plan of g. As before, player 2's peril is own-plan dependent. In this game P_2 depends both on player 2's plan of c or r, and of s or g. Initial peril is zero if player 2 plans s or if player 2 believes that player 1 plans to choose n. If player 2 plans s with some positive probability, then the initial peril is smaller than in the previous example, all other things being equal. As before, all negative outcomes follows a, and all outcomes following a are negative, but now a is conditional on g. Conditional on (g, a), P_2 corresponds to player 2's expected material payoff.

Player 2's updated peril if his or her second decision node is reached is

$$P_2\left((g,a)|\alpha_2\right) = 50\left(1-\alpha^r_{2,2}\right) + \left(500\left(1-\alpha^f_{2,1}\right) + 10\alpha^f_{2,1}\right)\alpha^r_{2,2}. \tag{10}$$

Player 2's updated peril is thus the same as in the previous example once that subgame is reached. The updated peril at (g, a) corresponds to player 2's expected material payoff at (g, a).

The increase in peril is

$$P_2'((g,a)|\alpha_2) = \left(1-\alpha_{2,2}^g\alpha_{2,1}^a\right)\times \left[50\left(1-\alpha_{2,2}^r\right)+\left(500\left(1-\alpha_{2,1}^f\right)+10\alpha_{2,1}^f\right)\alpha_{2,2}^r\right]. \tag{11}$$

If player 2 plans s with some positive probability, then the increase in peril is higher than in the previous example, should his or her decision node be reached. Player 1's optimal plan is still to maximize the probability of an attempt, conditional on it being sufficiently small such that player 2 transitions to the fearful state of mind after (g,a). Note that if player 2 plans s with some positive probability, then player 1 may, depending on player 2's fear threshold, plan a with certainty. Player 2's increase in peril at his or her second decision node comes from his or her own plan of choosing s. As before, player 1 plans f if player 2 chooses r.

Equilibria As before, when $\tau_2 > 50$, the game has a unique SE in $((n,f),(g,r))$, just as the standard game. Player 2 cannot transition to the fearful state of mind regardless of own and player 1's plan. As before, $((n,f),(g,r))$ remains an SE also when $10 < \tau_2 \leq 50$. For this intermediate range of τ_2, player 2's state of mind after (g,a) depends on his or her own plan. If player 2 plans (g,r), then he or she transitions to the fearful state if $P_2'(a|\alpha_2) = 10(1-\alpha_{2,1}^a) \geq 10$, which only occurs if $\alpha_{2,1}^a = 0$. Hence, if player 2 plans r, then he or she remains in the neutral state after (g,a), and maximizes own material payoff by choosing r. Player 1 maximizes own expected payoff by choosing (n,f). When $10 < \tau_2 \leq 50$, then $((a,f),(s,c))$ qualifies as another SE. If player 2 plans (s,c), then he or she transitions to the fearful state of mind after (g,a) if $P_2'((s,a)|\alpha_2) = 50 \geq \tau_2$. Thus, player 1 can guarantee a transition by planning a, and player 2 maximizes own minimum payoff by choosing c. Moreover, when $500/11 \leq \tau_2 \leq 50$, then there is an additional SE in $\left((g,c),\left(\left(\frac{\tau_2}{50},1-\frac{\tau_2}{50}\right),f\right)\right)$. Player 2 plans (g,c) and transitions to the fearful state after (g,a) if $P_2'((g,a)|\alpha_2) = 50(1-\alpha_{2,1}^a) \geq 10$. Player 1 can randomizes such that player 2 transitions by choosing $\alpha_{1,1}^a = 1-\tau_2/50$. Player 2 maximizes own minimum payoff after (g,a) by choosing c. In addition, player 2 is in the neutral state of mind at his or her first decision node and maximizes own expected material payoff by choosing g since $\alpha_{2,1}^a = 1-\tau_2/50$ is sufficiently small. Finally, when $\tau_2 \leq 10$, then $((n,f),(g,r))$ cannot be an SE.

To verify, assume it were. Player 2 plans (g, r), and transitions to the fearful state if $P_2' ((g, a)|\alpha_2) = 10(1 - \alpha_{2,1}^a) \geq \tau_2$. Player 1 can randomize between n and a such that player 2 transitions to the fearful state by choosing $\alpha_{1,1}^a \leq 1 - \tau_2/10$. Player 2 transitions to the fearful state after (g, a), and chooses c to maximize own minimum payoff, contradicting that $((n, f), (g, r))$ is an SE. When $\tau_2 \leq 10$ then $((a, f), (s, c))$ is the unique SE. If player 2 plans (s, c), then he or she transitions to the fearful state if $P_2' ((s, a)|\alpha_2) = 50 \geq \tau_2$. Thus, player 1 can guarantee a transition by planning a, and player 2 maximizes own minimum payoff by choosing c. □

In other words, if τ_2 is sufficiently large such that he or she cannot transition to the fearful state of mind regardless of own and player 1's plan, then the unique SE is identical to the SE in the game between players with standard preferences. For an intermediate range of τ_2, there is a multiplicity of SE. As before, because player 2's peril is own-plan dependent, his or her beliefs are self-fulfilling, and both $((n, f), (g, r))$ and $((a, f), (s, c))$ are SE. In addition, for a segment of the intermediate range there is a third SE in which player 2 plans g, c. Since the probability of a is sufficiently small, player 2 maximizes own material payoff by choosing g over s, aware of the probability of a and knowing that he or she will choose c conditional on a. Finally, if τ_2 is sufficiently small, such that player 2 may transition to the fearful state of mind regardless of own plan, then the unique SE is the one in which player 2 chooses s and forgoes the payoff from g.

Proposition 2. If τ_2 is sufficiently small such that player 2 can transition to the fearful state of mind after (g, a), then there is an SE in which $\alpha_{1,1}^a > 0$ and $\alpha_{2,2} = (s, c)$. Moreover, if τ_2 is sufficiently small such that player 2 can transition to the fearful state after (g, a) regardless of own plan, then the SE is unique.

Fear insensitive victims goes for a walk and receives the utility thereof, whereas fear sensitive victims stay inside. However, if the probability of a robbery attempt is sufficiently small, some fear sensitive victims may go for a walk while planning to comply if an attempt occurs. As before, this hinges on the fear thresholds being common knowledge. In games of incomplete information, the robber may find it difficult to distinguish between victims with different fear threshold.

Observations The analysis of the robbery game relies crucially on all player 2's payoffs following a robbery attempt being negative. More specifically, c is player 2's maximin action after (g, a) and c leads to a negative outcome. The maximin action leading to a negative outcome is necessary for fear to occur on the equilibrium path.

Consider the game in Figure 1. If player 2 believes that he or she will transition to the fearful state of mind and thus plans c, then he or she will indeed transition (if sufficiently fear sensitive) and thus c is an optimal plan. Player 2 becomes fearful on the equilibrium path.

The analysis changes if the maximin action leads to a non-negative outcome. A player who believes that they will become fearful and therefore optimally plans the maximin action will not experience an increased peril if the maximin action only leads to non-negative outcomes. Peril is zero and the player prefers to deviate to the action that maximizes the expected material payoff. Consider the game in Figure 3, in which the payoffs are normalized such that player 2's only negative payoff is when player 1 chooses v conditional on r. Further, there is an exogenous probability $\varepsilon > 0$ that player 1 trembles, and, by accident, chooses v.[19]

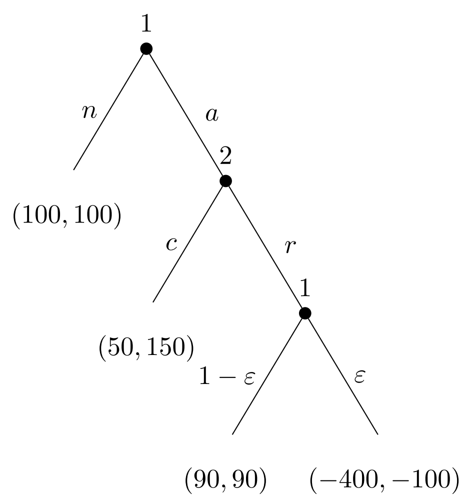

Figure 3: Robbery game with a single negative outcome.

In this game, a sufficiently fear sensitive player 2 cannot have an optimal plan. To see this, note that if player 2 plans c, then he or she does not transition to the

[19]Note that there is no increase in P_2 after a in this game unless there is some positive probability ε, that player 1 trembles and chooses v. Otherwise, both initial and updated peril is zero, as player 2, in equilibrium, never believes that player 1 plans v.

fearful state of mind after a, and maximizes own expected payoff by choosing r. If player 2 plans r, then player 1 can randomize such that player 2 transitions to the fearful state of mind after a, and player 2 then maximizes own minimum payoff by choosing c. In games where the maximin action does not lead to a negative outcome, the own-plan dependency of a player's peril can lead to self-negating beliefs. Further, for all values of τ_2, if player 2's decision node is reached, then he or she has strict preferences over the two actions and cannot have an optimal non-degenerate plan. Consequently, a sufficiently fear sensitive player 2, $\tau_2 \leq 400\varepsilon$, cannot have an optimal plan and no SE exists.

There are at least two possible solutions to the problem posed by this example. The first is to construct an equilibrium that is consistent with rationality constraints by smoothing the utility function around τ. The utility function u can thus be approximated by a continuous function u' which has a value equivalent to u's value accept in the boundary around τ where u is discontinuous. The second approach is to study ε equilibria (see e.g. Fudenberg & Levine, 1986; Jackson et al., 2012; Monderer & Samet, 1989; Radner, 1980). However, it is worth to keep in mind that emotions may pose a real and substantial obstacle for the existence of equilibrium.

There is one additional source of non-existing equilibrium in this model. Consider the case of a player 1 with an incentive to choose the probability of a as *small* as possible *without* causing player 2 to transition to the fearful state of mind. Since P_2' is decreasing in the probability of a and player 2 transitions to the fearful state of mind if $P_2' \leq \tau_2$, a player 1 with these incentives does not have an optimal plan and no equilibrium exists.

4 Bank runs

Fear can be of importance also when players' incentives are aligned. The game studied in this application is a multi-stage game between three personal players and chance. Note that the personal players are the bank clients. The bank is considered passive throughout the game. The game is a simplification of Diamond and Dybvig's (1983) seminal bank run game and is inspired by the experimental work of Garratt & Keister (2009).

The game proceeds in three stages. In stage 0 the players have 1 monetary unit and simultaneously decide whether to deposit it in the bank (d) or keep it in the mattress (k). Chance is passive in stage 0. In stage 1 and 2 of the game, all players with a deposit in the bank decide whether to withdraw their deposit (w) or let it remain in the bank (r). The player's decision tree is illustrated in Figure (4).

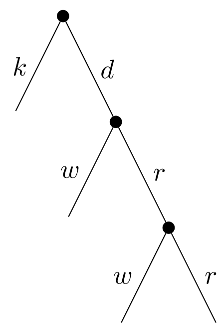

Figure 4: The personal players' decisions in the bank run game.

If a player decides to withdraw the deposit, the deposit is withdrawn with probability 1. In addition, the players face an exogenous risk of 'needing money tomorrow': in each stage each player faces the probability $\varepsilon \in (0, 1)$ of being selected by chance and forced to withdraw. I assume that the exogenous risk of withdrawal is independent both between stages and between players.

Let D denote the number of players who deposited their money in the bank in stage 0. After stage 0, the bank's assets equals the units deposited by the players. If $D \geq 1$, then the bank invests in a technology for an immediate cost of 1. This technology can provide a return after stage 2. The bank needs to liquidate the assets if the number of withdrawals in stage 1 and 2 is weakly greater than $D-1$. Let W_k, $k \in \{1, 2\}$ denote the number of withdrawals in stage 1 and 2 of the game. The bank liquidates the assets in stage 1 if $W_1 > D - 1$, and in stage 2 if $W_1 + W_2 > D - 1$. If the bank does not liquidate the assets in either stage, the technology 'bears fruit' and provides a monetary return.

The payoffs are normalized such that only losing the full deposit is considered a negative outcome. The normalized payoffs are illustrated in Table 1, conditional on all players depositing their money in stage 0. If no player withdraws their deposit, then they each receive a normalized payoff of 7. If only one player withdraws, then he or she receive a normalized payoff of 1 and the two players not withdrawing receive a normalized payoff of 5 each. If two players withdraw, then they each receive a normalized payoff of 1, while the player not withdrawing loses the full deposit and receives a normalized payoff of -1. This can occur either by the two players withdrawing in the same stage or by one of them withdrawing in stage 1 and the other in stage 2. If all three players withdraw their deposit, then their normalized payoffs depend on the order in which they withdraw. If all three players withdraw simultaneous, then they each receive a payoff of 2/3. If one player withdraws in stage 1 and the other two players in stage 2, then the player withdrawing in stage 1 receives a normalized payoff of 1 and the other two a normalized payoff of 1/2. The normalized payoff from not depositing the unit is 1.[20]

Table 1: Normalized material payoffs given that all players deposit their unit.

# Withdrawals	$\pi_i(r)$	$\pi_i(w)$
0	7	NA
1	5	1
2	-1	1
3	NA	2/3
1, 2	NA	1, 1/2

There is a multiplicity of SE in this game and the focus of this analysis is on a 'good strategy profile' in which all players plan to deposit their unit in the bank and not to withdraw in either stage.

Definition 3. The strategy profile σ is a *good strategy profile* if $\sigma_i = (d, r, r)$ for all $i \in I$.

Players with standard preferences Consider the game between players with standard preferences. If they use the good strategy profile, then their initially

[20]This application can be generalized to other payoffs. Crucial for the results is that only the outcome in which the player loses his or her full deposit has a negative material payoff. The other values are used to calculate and compare threshold values for ε for different equilibria.

expected payoff is[21]

$$\begin{aligned}\mathbb{E}\left[\pi_i\right|(3,r)\,;\alpha_i] =&(2/3)\varepsilon^3 + (1-\varepsilon)\varepsilon^2 + (1-\varepsilon)^2\varepsilon \\ &+ 2(1-\varepsilon)^2\varepsilon\mathbb{E}\left[\pi_i\right|((3,1),r)\,;\alpha_i] \\ &+ (1-\varepsilon)^3\mathbb{E}\left[\pi_i\right|((3,0),r)\,;\alpha_i]\,.\end{aligned} \tag{12}$$

When the players use the good strategy profile, they only withdraw their deposit if forced to do so. The first term is the (exogenous) probability that all players withdraw in stage 1 for a normalized material payoff of (2/3). The second term is the probability that two players withdraw. If player i is one of them, which can occur in two combinations, then he or she receives a normalized material payoff of 1, otherwise he or she receives a normalized material payoff of -1. The third term is the probability that only player i withdraws in stage 1 for a normalized material payoff of 1. The fourth term is the probability that one (other) player withdraws in stage 1 times the expected material payoff from stage 2. Likewise, the fifth term is the probability that no player withdraws in stage 1 times the expected material payoff from stage 2.

At the beginning of stage 2, the players observe the withdrawals in stage 1. If two or more players withdrew, then a bank collapse has occurred and the remaining player (if any) loses all of his or her money. If one player withdrew, then the expected payoff from not withdrawing in stage 2 is

$$\mathbb{E}\left[\pi_i\right|((3,1),r)\,;\alpha_i] = 5(1-\varepsilon)^2 + (1/2)\varepsilon^2. \tag{13}$$

If no player withdrew, then the expected payoff from not withdrawing in stage 2 is

$$\mathbb{E}\left[\pi_i\right|((3,0),r)\,;\alpha_i] = (2/3)\varepsilon^3 + (1-\varepsilon)\varepsilon^2 + 11(1-\varepsilon)^2\varepsilon + 7(1-\varepsilon)^3. \tag{14}$$

The payoff from deviating and withdrawing the deposit is 1 in either stage. For $\varepsilon < 0.496$ the *good strategy profile* is an SE when the players have standard preferences.

[21] With a slight abuse of notation, the history is written as the number of deposits in stage 0 and the number of withdrawals observed after stage 1.

Players who may transition to the fearful state of mind Now consider players who may transition to the fearful state of mind. The maximin action in stages 1 and 2 is to withdraw the deposit. Assume the players have identical fear thresholds, τ_i, and that they use the good strategy profile. In this case, peril is due to the exogenous risk of withdrawal.

Player i's initial peril, the probability that the other two players will be forced to withdraw, is

$$P_i\left(h^0|\alpha_i\right) = (1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^3\varepsilon^2 + (1-\varepsilon)^4\varepsilon^2. \tag{15}$$

The first term is the probability that the other two players withdraw in stage 1. The second term is the probability that one of the other players withdraws in stage 1 and the other in stage 2. The third term is the probability that both players withdraw in stage 2.

If no player withdrew in stage 1, then the updated peril is

$$P_i\left((3,0)|\alpha_i\right) = (1-\varepsilon)\varepsilon^2. \tag{16}$$

The updated peril corresponds to the probability that both other players are forced to withdraw in stage 2, but not player i.

The updated peril is smaller than the initial peril and $P_i'((3,0)|\alpha_i) = 0$. There is no increase in peril and the players do not transition to the fearful state of mind. If at least 2 players were forced to withdraw, then the bank has collapsed. Any remaining player transitions to the fearful state of mind if sufficiently sensitive to fear, but he or she has no action left to take and receives a payoff of -1.

The more interesting case is when one player withdraws in stage 1. The updated peril for the remaining players is

$$P_i\left((3,1)|\alpha_i\right) = (1-\varepsilon)\varepsilon. \tag{17}$$

The updated peril corresponds to the probability that (only) the other player is forced to withdraw in stage 2.

The increase in peril is

$$P_i'((3,1)|\alpha_i) = (1-\varepsilon)\varepsilon(1-(3(1-\varepsilon)^3+1)\varepsilon). \tag{18}$$

Let $\overline{\tau}$ denote the maximum fear threshold which allows for a transition to the fearful state of mind, $\overline{\tau} = (1-\varepsilon)\varepsilon(1-(3(1-\varepsilon)^3+1)\varepsilon)$. After observing one withdrawal in stage 1, the remaining players transition to the fearful state of mind if they are sufficiently sensitive to fear, $\tau_i \leq \overline{\tau}$. Once fearful, the players prefer to deviate to w. The plan of choosing r in both stages regardless the outcome of stage 1 can therefore not be an optimal plan for such players.

Proposition 3. If the players are sufficiently sensitive to fear, $\tau_i \leq \overline{\tau}$, then the *good strategy profile* cannot be an SE.

Fear sensitive bank run players may experience a welfare loss.[22] Moreover, this welfare loss is also imposed on their fear insensitive co-players. The social welfare maximizing strategy profile that fear sensitive players can coordinate on is not to withdraw in stage 1 and withdraw in stage 2 conditional on one player being forced to withdraw in stage 1. This strategy profile is an SE for $\varepsilon \leq 0.422$. In this SE no player is fearful on the equilibrium path. The players coordinate on withdrawing conditional on observing one withdrawal in stage 1, and withdrawing is not a negative outcome.

When the exogenous probability of withdrawal is sufficiently high, the game has a unique equilibrium in which all players chooses k and no money is invested. When the players may transition to the fearful state of mind, the exogenous probability for this equilibrium to be unique is smaller than for players with standard preferences, $\varepsilon > 0.422$ compared with $\varepsilon > 0.479$. Since fear sensitive players cannot commit to r conditional on observing another player withdrawing, they are less willing to choose d to begin with. Fear sensitive players thus experience a welfare loss compared with players with standard preferences for intermediate values of the exogenous risk.

Proposition 4. For $\varepsilon \in (0.422, 0.479]$, fear causes a welfare loss to the players if, for some $i \in I$ $\tau_i \leq \overline{\tau}$.

Notice that players who are insensitive to fear also experience this welfare loss when interacting with fear sensitive players. Consider the situation with

[22]Defining welfare for emotional players is a non-trivial issue. Behavioral welfare economics is a field of its own (see e.g. Bernheim & Rangel, 2009). In this analysis I focus on the welfare experienced by the player in the neutral state of mind. In other words, the player's monetary payoff.

one sufficiently fear sensitive players and two fear insensitive player. If one of the fear insensitive players is forced to withdraw in stage 1, then the remaining fear sensitive player prefers to withdraw. Hence, it is in the best interest also for the fear insensitive player to withdraw in stage 2 conditional on observing one withdrawal in stage 1, conditional on knowing that the other player's fear threshold has been reached.

5 Public health intervention

Finally, consider a simple example of a public health authority who wants to inform the public about accurate estimates of the probability or cost of contracting a disease. This example stem from the observation that in many cases an individual's beliefs over the probability or cost of a negative outcome are mistaken.[23]

Consider a decision maker, player 1, who contemplates whether to take a vaccine against a disease. If player 1 takes the vaccine, then he or she becomes immune to the disease. Otherwise he or she faces a positive probability of contracting it. Player 1 holds correct beliefs over the probability of contracting the disease and the cost of vaccination. However, player 1 has underestimated the cost of the disease.[24] Recognizing this, the public health authority launches an information campaign to inform player 1 about the true cost.[25] Note that the public health authority is not considered a player in this scenario.

The probability of contracting the disease (if not vaccinated) is denoted by $\varepsilon > 0$, and the cost of vaccination is denoted by v, $0 < v < 1$. Player 1's prior belief over the cost of the disease is denoted by d, and his or her updated belief is denoted by d', where $d' > d > 1$.

The payoffs are normalized such that the only negative outcome is to contract the disease. The normalized payoff from not taking the vaccine and not contracting the disease is 1. The normalized payoff from taking the vaccine is $1 - v$. Finally, the normalized expected payoff of contracting the disease is $1 - d$ and $1 - d'$, when player 1 is uninformed and informed, respectively.

[23] Or they may be correct but still possible to influence.

[24] A related situation is the more ethically dubious case when the authority wants to scare people into a certain behavior by making them overestimate the cost.

[25] This example is inspired by a recent Australian government Covid-19 awareness advertisement showing a young woman in distress in a hospital bed that has been criticized for leaning into scare tactics (BBC, 2021).

First consider a player 1 who initially plans to take the vaccine. The player is risk neutral in the neutral state of mind and takes the vaccine if $v \leq \varepsilon d$. Because the only negative outcome is contracting the disease, player 1's initial peril is zero, $P_1(h^0; \alpha_1) = 0$. He or she will take the vaccine and become immune to the disease. The public health authority informs player 1 about the correct cost of the disease. However, since player 1 plans to take the vaccine, the updated peril is zero. There is no change in peril and, since player 1 planned to take the vaccine regardless, player 1 is unaffected by the information.

Next consider a player 1 who initially plans not to take the vaccine. That is, $v \geq \varepsilon d$. Player 1's initial peril is

$$P_1(h^0; \alpha_1) = \varepsilon \times d. \tag{19}$$

The public health authority informs player 1 about the correct cost of the disease. As in traditional theory, player 1 changes his or her plan of taking the vaccine if $c \leq \varepsilon d'$. In addition, a fear sensitive player may transition to the fearful state of mind if the information causes a sufficiently large increase in peril.

Player 1's increase in peril is

$$P_1' = \varepsilon(d' - d). \tag{20}$$

Player 1 transitions to the fearful state of mind if the updated cost of the disease is sufficiently large such that $\tau_1 \leq P_1'$. In the fearful state of mind, player 1 maximizes minimum payoff and takes the vaccine also when $v > \varepsilon d'$. In other words, player 1 will take the vaccine not only for higher values of c,but also when the costs outweigh the benefits.

Proposition 5. Fear sensitivity can amplify the behavioral response to information that increases the expected cost of negative outcomes.

As in traditional theory, if $v \in (\varepsilon d, \varepsilon d']$, then information alters behavior. A player who remains in the neutral state of mind after receiving the information takes the vaccine also when $v > \varepsilon d'$. However, a player who transitions to the fearful state of mind after receiving information takes the vaccine regardless the value of v. Consequently, fear sensitive players with a high cost of vaccination may overconsume the vaccine.[26] At the same time, vaccines often have a positive

[26]The cost of vaccination may for example include expected side effects that may vary between

externality. For example if the disease is highly transmissible and people fail to take that into account. In such a situation a fear response may be desired by a public health authority to increase social welfare.

6 Concluding Remarks

This paper presents a model of players who can transition from a neutral to a fearful state of mind. In the neutral state of mind, the player maximizes own expected material payoff. In the fearful state of mind, the player maximizes own minimum material payoff. The players are in the neutral state of mind when the game begins and transition to the fearful state after a sufficiently large increase in the expected cost of negative outcomes, the outcomes bad enough to potentially instill fear when anticipated.

Fear has various effects on behavior. The focus of this paper is on how fear affects behavior through it's effect on a player's risk aversion. Fear can also be unpleasant. Psychology research distinguishes between an emotions valence, it's direct effect on utility, and it's action tendency, it's direct effect on behavior. While the negative valence of fear is an incentive for players to avoid fearful situations, I abstract from this in this paper to focus on the direct behavioral consequences of fear. Fear is a state of mind which strongly influences decision making and it is not uncommon for people to experience fear.

The transition from the neutral to the fearful state of mind is determined by beliefs over outcomes. This definition rules out some belief-dependent triggers of fear such as someone who becomes fearful if he or she believes that their boss believes that he or she has low productivity. However, one can model this situation as a player with the negative outcome of losing his or her job. An increase in the belief that the boss believes that he or she has low productivity causes an increase in the probability that he or she will lose the job, which may trigger fear.

Players' increased concerned with risk when they have transitioned to the fearful state of mind causes an amplified response that might seem like an overreaction. This observation is in line with empirical and experimental research that finds a 'residual' change in behavior after bad news or adverse events (Guerrero

DMs of different health status.

et al., 2012; Guiso et al., 2018; Piccoli et al., 2017; Wang & Young, 2020). In other words, the behavioral response is stronger than what can be explained by traditional factors alone. The residual change in behavior is however consistent with an emotion-based change of the utility function.

In this paper I restrict my focus to sophisticated players who can perfectly predict own and others' state transitions and the behavioral consequences thereof. In the sequential equilibrium for psychological games, the players are certain about own and others' plans, and never change their minds about them. Any deviations from the plans are interpreted as mistakes. This is a strong assumption, especially since the players' decision utility may depend on both own and others' plans. However, the analysis shows how fear is of importance even when players can perfectly predict their own state transitions. Whereas the study of naïve players who cannot perfectly predict own emotional response is outside the scope of this paper, it is certainly an interesting avenue for future research.

It is natural to ask whether fear, modeled as belief-dependent risk aversion, makes sense from an evolutionary perspective. Aumann (2019) argues that people use rules to guide their behavior and that these behavioral rules are the product of evolutionary forces. Rather than maximizing utility over actions, people adopt rules of behavior that do well in usual, naturally occurring situations. A simple example of such a rule related to fear could be 'maximize expected utility when in a safe environment while minimize the worst losses when in peril'.

Besides robberies, bank runs, and public health policy, fear may be important in understanding for example conflict; environmental dangers and climate risk; and financial decision making. These applications are potentially of great importance and are left as an avenue for future research.

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