The importance of instruction in mathematical classrooms with increasing diversity of language.

A model for description and analysis of instructional modes.

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In this paper the importance of teacher responsibility for knowledge generation in mathematical classrooms with increasing number of students with diverse language backgrounds is discussed. A model for description and analysis of important dimensions of teacher responsibility in instructional practise is proposed. This model challenges the potential of the traditional division into teacher- versus student-centred modes of instruction. To justify the validity of the suggested model it was confirmed against Swedish data from TIMSS 2003. The empirical results supported the possibility to adopt this alternative perspective on modes of instruction.

BACKGROUND

European Union is undergoing considerable demographic change with diverse language groups moving into Europe and also across national borders within the Union. The instructional practise in mathematics’ education is challenged by this increasing diversity of language in the classrooms.

As indicated by TIMSS-studies, Swedish students have shown a general decline in mathematics achievement from 1995 to 2007 which is valid both for students with Swedish and foreign background. Second-language students in the compulsory school are however less successful in mathematics than their Swedish classmates and the proportion of students without certificates in mathematics is larger among these students (Skolverket, 2008). Mathematical test results with low averages and increasing dispersion, are for Sweden similar to other countries’ with an early differentiation (Hanushek & Wössmann, 2006). Mathematics education is however formally untracked in Sweden. This calls for research about effects of background factors on student’s mathematics achievements, specifically those factors that have changed during the period of decline. The socioeconomic segregation in the Swedish schools has continued to increase in the 21st century (Gustafsson, 2006), and this probably effects the performance. But it is also established that during the last two decades mathematics education has in Sweden become more and more individualized with less interaction among participants and less instruction and teacher responsibility for knowledge generation in favour of more “student’s independent work” (Skolverket, 2004; Skolverkets Rapport nr. 323, 2008; Vinterek, 2006).
This calls for research about effects for second language students’ performance in mathematics of, not only the socioeconomic status, but also the individualization and diminishing teacher responsibility for the knowledge generation. For this reason a model for description and analysis of mathematics instructional practise, that has the potential to highlight important dimensions of teacher responsibility for knowledge generation and how this responsibility is expressed in the instructional practise, is needed.

**Second language students and mathematics learning.**

There are previous research saying that for second language students’ mathematical progress it is not of vital importance if they learn mathematics on their second language or not, e.g. (Clarkson, 2006; Davidenko, 2000; Jäppinen, 2005), while other research shows that multilingualism in many ways is an obstacle or a complication in mathematics education, e.g. (Barton & Neville Barton, 2003; Secada, 1992; Setati, 2002, 2005; Setati & Adler, 2000). In the prior studies it is shown that second language students with well developed language skills in both their language perform as well or better than other students in mathematics. Those results could be related to the theory of threshold by Cummins and Swain (1986) where it is stated that it is of importance to have well developed skills in both languages. These results motivates to further examine the significance of diverse dimensions in the mathematical instructional practise related to students’ language use.

From a Vygotskian holistic point of view, learning is understood as a process where more complex structures of knowledge, present in the surrounding world, could be attained through interaction with other people (Vygotskij, 1978). The social action is to be viewed as a precondition for the individual action. The way participants in the mathematics classroom use different tools, as for instance language, for interpersonal and intrapersonal communication will affect the knowledge generation (Vygotskij, 1986). From this point of view teachers thus have the responsibility for arranging the instructional practise in order to enable this communication and thinking. Gibbons (2006) describes this model for learning as an alternative learning model with interaction between teachers and students and that both have an active role in the learning process. In this model the meanings of words and concepts are viewed as individual contractions (Barwell, 2005; Säljö, 2000) where the language assumes to be dependent on students experiences and carries cultural traditions. Thus, the authors means, this has to be payed attention to in the mathematics education.

In one of two other often practised models, ”the banking model”, the teacher has the role of putting skills and knowledges on students empty memory banks. Here the teachers’ only mission will be to transfer knowledges. In the other model, ”the progressive model” which is supported by Dewey’s and Paget’s research, the students are not viewed as passive receivers but they are supposed to be active in asking questions and thinking clever thoughts. The mission of the teacher will here be to provide accurate working materials. In ”the banking model”, Gibbons (2006) states, one of the most important principles in language acquisition is opposed, the principle to use the language in interaction with others. She also means that the contextualisation of the mathematics content often
is relayed to the majority group of students in the classroom. Also the “progressive” model has shortages concerning the use of language and interaction, according to Gibbons, and in both models the students are viewed as independent individuals and the learning as something that occur in the individual. In those models words and concepts are to be viewed as unambiguous. Pimm (1995) could be regarded as an upholder of this model.

Cooper and Harries (2002; 2005) also problematise that mathematics education relayed to student’s previous experiences could treat second language students unfairly. Independent of which learning model that is present in the learning process, students from higher social groups in the society, well familiar with the context present in everyday situations and communication used in the classroom, often are favoured, the authors states. This could indicate that students with foreign background not automatically proceed in mathematics when communication is apparent. Moschkovich (2002) however argues that all manners of communication and talking could contribute in its own way to the mathematical discussion and bring resources to the conversation, thus she propose interaction in the mathematics education. To avoid stereotype apprehensions about second language learners ability to perform as well as other students, it is of importance in mathematics education to regard that the education not often is culture- and language neutral (Hyltenstam, 1993; Parszyk, 1999). Also Bruner (2002) points out the importance of education to be viewed in its cultural context.

All together, much indicates that it is of importance not only what skills multilingual students have in their languages but also how they make use of their languages when learning mathematics (Cummins, 1984; Setati & Adler, 2000). This could be viewed as aspects of the theory of scaffolding which means that the teacher has the responsibility for helping students to reach new skills, new concepts and new levels of understanding (Vygotskij, 1978). For second language learners it thus is crucial to use language in cognitive and linguistically challenging situations (Gibbons, 2006). In e.g. a study by Barwell (2003) second-language students are shown to be possible to participate productively in a mathematical classroom task, when the task supports the use of the second language and contributes to the mathematics.

According to Collier and Thomas (2002) it will take five to ten years in a rich language environment for children to receive satisfactory capacity in the second language for fully following the instruction, and about two years to receive a basis vocabulary. Cummins (2000) has suggested that it takes between one and two years for a second language learner to develop “context-embedded” (everyday) language and between five to seven years to develop “context-reduced” (academic register of schooling) language. This could be interpreted as mathematics instruction not to have that much meaning for second language student’s opportunity to learn mathematics. But some studies shows that if second-language students from the teacher get explicit explanations and rules for words and concepts and if those explanations are coherent with the implicit language use, these students are given a better chance to learn mathematics (Elmeroth, 2006; Jamieson, Chapelle, & Preiss, 2004). Gibbons (2002) means:
Ultimately, if second language learners are not to be disadvantaged in their long-term learning, and are to have the time and opportunity to learn the subject-specific registers of school, they need access to an ongoing language-focused program across the whole curriculum. (p. 5)

This underlines the importance of teacher responsibility for letting language and communication being an active part in the instruction and students’ mathematical progress. For language teaching there is a model developed by Cummins (1984) that makes a note of the interplay between cognitive degree of difficulty and context-reduced language. By scaffolding teachers have the responsibility to support second language students’ progress from low cognitive level and context-embedded language towards more high cognitive levels where language is used as a mental tool for thinking. This model should probably be significant also in mathematics teaching. Language and understanding will be constructed in a mutual process between the more and the less competent speaker, where the cooperation is characterized by negotiation with foundation for interpreting given by the second language learner (Lindberg, 2004).

Research in South Africa is characterized by the Vygotskian holistic point of view, (Vygotskij, 1978). Code switching, which means that the teaching is realized alternately on different language in the classroom (Setati & Adler, 2000), is one of many conditions studied. Outside South Africa this concordance in research results is not present. Some studies states that it is most successful to teach on students first language, e.g. (Ellerton & Clarkson, 1996), while others, e.g. (Secada, 1996), stress the importance to use all language skills when learning an academic subject.

To sum up, with theoretical starting points in a theory of learning where interaction between teachers and students are central and where it is of significance that both have an active role in the learning process, it is of importance to pay attention to the teacher responsibility for the knowledge generation and for second-language student’s language use in the mathematics education. Teachers have the responsibility for arranging the instructional practise in order to enable communication and thinking and to confront second language students to use language in cognitive and linguistically challenging situations. Teachers also have the responsibility for giving explicit explanations and roles for words and concepts.

**How to conceptualise and define class-room practise.**

The term *Instructional modes* is in this study used as a label to depict what implicit pedagogical principles underlie classroom practises of instruction and teacher responsibility for mathematics knowledge generation. They are viewed as potentially important for second language students’ opportunity to learn mathematics. This demands development of a model for description and analysis of these modes. Conceptualisation of important dimensions of classroom practise supportive for second language student’s mathematical progress will constitute the theoretical starting-points when developing such a model. From previous research dimensions that deal with teacher responsibility and its consequences will be discussed.
It has been argued that instructional modes affording opportunities for interaction, talking and improving language skills are supportive of second language student’s mathematical progress (Vygotskij, 1986). Teacher has the responsibility for framing the instructional practice for this learning environment which could be viewed as an aspect of the theory of scaffolding (Vygotskij, 1978). Also Cummin (1984) emphasizes the meaning of teachers responsibility for scaffolding in his model for language teaching. For using language in cognitive and linguistically challenging situations (Gibbons, 2006) it also implies that teachers take the responsibility for affording such opportunities. It has also been argued that teachers have the responsibility for giving explicit explanations and roles for words and concepts.

The theoretical framework in a study by Clarke and Xu (2008) concerns the distribution of responsibility for the knowledge generation between teacher and students, but in that study it is focused on students’ and not on teacher’s part of the responsibility. Teacher’s part seems in that study to be taken for granted. Most previous research however concerns the nature of or effects of implicit teacher responsibility. Conversation and interaction as an important dimension of the instructional practice is maintained by e.g. (King, 1992; Weber, Maher, Powell, & Stohl Lee, 2008; Yackel, Cobb, Wood, Wheatley, & Merckel, 1990). Another qualitative aspect of interaction and talking is in which way teachers make use of the variation between students and how they take charge of the peer effects (Barwell & Clarkson, 2004; Hanushek & Wössmann, 2006; Shayer & Adhami, 2007).

A widespread way to characterise mathematics education is to distinguish between teacher- and student-centred modes of instruction. The ”traditional instructional mode” is characterised by the teacher-centred instruction where the teacher mainly explains procedures and gives directions (Hiebert, et al., 2003). The students are expected to listen and remember what the teacher says and very little time is spent on letting the students explain thoughts and reach consensus about mathematical ideas. Interaction is here not supposed to be prominent. This category has however been challenged in previous research by exposing students interaction and active thinking to be present despite teacher-centred instruction (Clarke, 2006; Häggeström, 2008; Mok, 2003).

The student-centred instruction is likely to be characterised more by interaction in learning, which is meant to develop the mathematical identity of the student (Ball & Bass, 2000; Boaler & Greeno, 2000). The teacher is in this instructional mode viewed as vital for initiating the interaction between participants and to strive for high quality in the conversation (Yackel, et al., 1990).

Instructional modes in the Swedish mathematics education is often characterized by student-centred modes due to “student’s independent work” with sparse interaction, communication and teacher intervention (Skolverkets Rapport nr. 323, 2008; Vinterek, 2006). Sweden, a country with no early tracking and with expected student-centred instruction, shows astonishing results of mathematics achievements with low averages and increasing dispersion (Hanushek & Wössmann, 2006), and this implies further analyses of contextual background factors. Since the essential dimensions of
underlying pedagogical principles in the Swedish mathematics education are not necessarily uncovered by the traditional way of categorizing instructional modes as teacher- or student-centred, an alternative model for description and analysing of instructional modes is required. Dimensions of teacher responsibility for the knowledge generation and for second language students language use in the mathematics education framed in a theory of learning where interaction between teachers and students are central and where it is of significance that both have an active role in the learning process, should be included in such model.

The aim of this study is to develop such a model for description and analysing of instructional practise. When applied in further studies, the model should have the potential to clarify how instruction does effect mathematics achievements for second language students. To justify the validity of the developed model it will in this study be confirmed against data from TIMSS 2003 8th grade. Sweden will be used as an example.

**METHOD**

The empirical study was realized as a secondary analysis of TIMSS data from 2003 focusing on mathematics for Swedish students in 8th grade. In an ongoing study, which will be presented later, the analysis is being conducted on data from 2007. A latent variable analysis was conducted in order to identify descriptive dimensions of instructional modes concerning both teacher responsibility and mathematics content which may support student’s mathematical progress.

**Data Sources**

The data source for the empirical study was the TIMSS 2003 study, focusing on mathematics for Swedish students in 8th grade, with 4 256 students from 274 classes in 160 schools. The contextual variables were derived from teacher and student questionnaires. In the data subset used, only those classes with one mathematics teacher were included (253 classes). After listwise deletion, there were 3 288 observations left in 217 classes with an average cluster size of 15.15. For items used in the analysis, see Table 1.

**Latent variable analysis through multilevel confirmatory factor analysis, M-CFA**

Because of the design effect in survey research using cluster samples it was necessary to take into account the hierarchical structure of the data (Hox, 2002). Therefore Multilevel Confirmatory Factor Analysis (M-CFA) was used as the method of analysis. CFA requires a strong empirical or conceptual foundation to guide the specification and evaluation of the factor model (Brown, 2006). A two-level structural equation model approach with three factors was adopted for the measurement model and the Mplus (Muthén & Muthén, 1998) and STREAMS (Gustafsson & Stahl, 2004) software was applied in the analyses. The Within-level represents individual students within the classes and the Between-level represents classes and concerns differences between classes. The latent factors were indicated by ten items from the student questionnaire and seven from the teacher questionnaire see Table 1. Students in the TIMSS study responded to the questions “In math
lessons, how often do you do…?” Responses were indicated on a 4-point Likert scale, which ranged from 1 (Every or almost every lesson) to 4 (never). Teachers either responded to the questions “In math lessons, how often do you ask students to…” or to “Which part of the lesson-time in mathematics does students…?” Responses were noted on a 4-point Likert scale, which ranged from 1 (Every or almost every lesson) to 4 (never), or by supplying a percentage estimate.

Several indices were used to assess model fit; *chi-square test*, *root-mean-square error of approximation* (RMSEA) and the *standardized root mean-square residual* (SRMR) (Brown, 2006). RMSEA values less than 0.05 represent a “close fit”, and models with values above 0.1 should be rejected. The SRMR was used as an absolute fit index. The SRMR value should be 0.08 or less. Because the chi-square statistic is very sensitive to sample size, chi-square/df ratio was examined to check fit (Kline, 1998). A goodness-of-fit index, *Comparative Fit Index*, CFI, shows a good performance overall (Hox, 2002). Usually a value of at least 0.95 is required to accept a model.

**Hypothesised measurement model for instructional modes**

In order to develop a measurement model with potential to highlight dimensions of teacher responsibility for knowledge generation and how this responsibility is expressed in the instructional practise, the theoretical starting-points for the number of latent factors and appropriate indicators were made explicit. It was hypothesised that modes sustaining both teacher responsibility for emphasizing and preparing the mathematics content and teacher responsibility for making second language student’s experiences and reasoning about the content visible in a way that enables interaction between teachers and students and where both have an active role in the learning process, are characteristics of such approaches (Cummins, 1984; Gibbons, 2006; L. S. Vygotskij, 1978). These two dimensions of underlying important pedagogical principles for instructional modes were in the model hypothesized to be represented by two latent factors labelled *Teacher Responsibility for Content (TRC)* and *Student Responsibility for Learning (SRL)*. It also was hypothesised that instruction which aims to develop the students previously acquired knowledge and which is focusing on conceptual instead of mostly procedural mathematics may support student’s mathematical progress (Lester, 2007; Lithner, 2006). Hence a third descriptive dimension, *Challenging Mathematics Content (CMC)*, dealing with such educational content that supports these aims was included in the model.

When mirrored against Swedish conditions, the dimension concerning teacher responsibility for emphasizing and preparing the mathematics content, *TRC*, was hypothesised to be an important underlying pedagogical principle. In Sweden the instructional mode “student’s independent work” is present in mathematics education and this practise is characterized by students both planning and working on their own with different tasks independent of other students, with responsibility laid on student’s individual liability for the mathematics learning (Ståhle, 2006; Vinterek, 2006; Österlind, 1998). It is also established that this instructional mode has increased during the last two decades (Skolverket, 2004; Skolverkets Rapport nr. 323, 2008). This dimension could be viewed as a
presumption for the mathematics content to be in focus and for interaction and talking to be realized, which in this study was hypothesised to be characteristics of approaches favouring second language student’s mathematical progress.

Observed variables from both the student- and the teacher questionnaires in the TIMSS-data were selected for indicating the latent factor TRC, see Table 1. From the student questionnaire, three observed variables were chosen to capture this latent factor, namely BSBMHLS (In your math lessons, how often do you listen to the teacher give a lecture-style presentation?), BSBMHHQT (In your math lessons, how often do you have a quiz or test?) and BSBMHWPO (In your math lessons, how often do you work problems on your own?). From the teacher questionnaire it was hypothesised that four observed variables have the capacity to reflect this latent factor, namely BTBMASWF (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to work fractions and decimals?), BTBMPTLS (In a typical week of mathematics lessons for the TIMSS class, what percentage of time do students spend listening to lecture-style presentations?), BTBMPTTQ (In a typical week of mathematics lessons for the TIMSS class, what percentage of time do students spend taking tests or quizzes?) and BTBMASPC (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to practice computational skills?). All the selected observed variables depict a classroom in which the teacher emphasizes and prepares the mathematics content. The manifest variable BSBMHWPO is to be viewed as an indicator concerning the individual student’s experiences of working with the mathematics content itself, rather than an indicator concerning their experiences of working autonomously. However, it was hypothesised that this manifest variable should not to be included on the class-level because it does probably not discern classes, depending on the customary presence of autonomously working in the classes. To sum up, this latent factor was included in the model aiming to show to which extent the responsibility for emphasizing and preparing the mathematics content is situated to the teacher. From the student- and teacher-questionnaires in the TIMSS data, seven observed variables were hypothesised to have the capacity to depict this latent factor.

For the second latent factor, SRL, indicators were selected which reflect to what extent teachers take the responsibility for making second language student’s experiences and reasoning about the content visible in a way that enables them to take responsibility for their own learning process, see Table 1. From the student questionnaire in the TIMSS-data four observed variables were hypothesised to have the capacity to indicate this latent factor, namely BSBMHMDL (In your math lessons, how often do you relate what you are learning in mathematics to your daily life?), BSBMHROH (In your math lessons, how often do you review your homework?), BSBMHEXP (In your math lessons, how often do you explain your answers to the class?) and BSBMHSCP (In your math lessons, how often do you decide on your own procedures for solving complex problems?). These observed variables mirror a classroom where student’s experiences and ways of reasoning about mathematical issues are made visible, i.e. teachers who use their responsibility for giving the students opportunities to relate to their every day life, to their experiences and to their work with
mathematical issues. From the teacher questionnaire the indicators BTBMASDL (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to relate what they are learning in mathematics to their daily lives?), BTBMHDAD, (How often do you use the homework as a basis for class discussion with the mathematics homework assignments?) and BTBMASEA (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to explain their answers?) were selected. BSBMHSCP is for the same reasons as those stated above for BSBMHWPO to be viewed as an indicator depicting diversity among students within classes but not between classes. To sum up, this latent factor, SRL, was included in the model aiming to show how teachers take the responsibility for making student’s experiences and reasoning about the content visible in a way that enables them to take responsibility for their own learning process. From the student- and teacher-questionnaires in the TIMSS data, seven observed variables were hypothesised to have the capacity to depict this latent factor.

Finally the latent factor labelled CMC, concerning mathematics content with high relevance and conceptual orientation was included in the model. From the student questionnaire in the TIMSS-data four observed variables were hypothesised to have the capacity to depict this latent factor, namely BSBMHASM (In your math lessons, how often do you practice adding, subtracting, multiplying, and dividing without using a calculator?), BSBMHWFD (In your lessons, how often do you work on fractions and decimals?) and BSBMHEFR (In your math lessons, how often do you write equations and functions to represent relationships?). This mathematics content is supposed to be challenging to students’ previous skills, understanding and conceptions in letting them work with conceptual practises, work without using calculators and in stimulating students to create mathematical models by themselves. This content does also emphasize mathematic subjects with high relevance for student’s further mathematical progress.

Table 1: List of Items used in the CFA, Within(individual)- and Between(class)-level

<table>
<thead>
<tr>
<th>Item</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TRC</td>
<td>SRL</td>
<td>CMC</td>
</tr>
<tr>
<td>BSBMHLSP; In your math lessons, how often do you listen to the teacher give a lecture-style presentation?</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>BSBMHHQT; In your math lessons, how often do you have a quiz or test?</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>BSBMHWPO; In your math lessons, how often do you work problems on your own?</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSBMHDML; In your math lessons, how often do you relate what you are learning in mathematics to your daily life?</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>BSBMHRROH; In your math lessons, how often do you review your homework?</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>BSBMHEXP; In your math lessons, how often do you explain your answers to the class?</td>
<td>x</td>
<td>x</td>
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**RESULTS**

In order to evaluate the potential for this hypothesized model to work as an instrument on empirical data, an examination of the correspondence between the model and Swedish data from TIMSS 2003, 8th grade, was done. Results are shown in Figure 1, with factor loadings and variances for the indicators, both for the Within(individual)- and the Between(class)-level.

The intra-class correlation (ICC) suggested sizeable class effects, the ICCs ranging between 0.051-0.291. The model showed reasonable good fit. The comparative fit index CFI was 0.863 and the RMSEA was 0.036. There was an appropriate model-fit on the student level (SRMR = 0.027) whereas the fit on the class-level was somewhat more hard to interpret, the SRMR measure being somewhat higher (0.126) than the suggested criterion. As the modification indices on the Between-level showed no indications of local misfit, the impressions of poor model fit signalled by the SRMR index may be due to limitations of this index when applied in multilevel SEM (Brown,
The substantial meaningfulness of the model, and the possibilities of interpretation, will also contribute to the evaluation of the fit.

Notes. (Two-tailed Est./S.E.<2.0, P-value>0.05)

Figure 1. Factor loadings for Within(individual)- and Between(class)-level estimates

The latent variables were all positively correlated, but no correlation was higher than 0.8, which supports the hypothesis that the latent factors represent different construct (Brown 2006). See table 2.
Table 2: Factor correlation, Within- and Between-Level

<table>
<thead>
<tr>
<th></th>
<th>Factor 1, TRC</th>
<th>Factor 2, SRL</th>
<th>Factor 3, CMC</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Within-Level</td>
<td>Between-Level</td>
<td>Within-Level</td>
</tr>
<tr>
<td>Factor 1, TRC</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Factor 2, SRL</td>
<td>0.781</td>
<td>0.772</td>
<td>1.000</td>
</tr>
<tr>
<td>Factor 3, CMC</td>
<td>0.702</td>
<td>0.551</td>
<td>0.511</td>
</tr>
</tbody>
</table>

All factor loadings in the model were statistically significant. Substantially they also correspond to the underlying theoretical starting points for the model. For the latent factor TRC, which was hypothesised to be indicated by items mirroring to what extent teachers do take the responsibility for emphasizing and preparing the mathematics content, most factor-loadings for the selected indicators were significant and medium high, both on the Within- and on the Between-level. Two factor-loadings at the Between-level were however negatively loaded, and one of these was hardly significant. These indicators could still be interpreted as utterances from teachers who are taking the responsibility for knowledge generation. If the teacher during the mathematics lessons experience time being too limited for both traditional lectures, tests, probations and other pedagogical practices, then negative relations between these different categories of indicators could arise. The scaling for the two negatively loaded indicators also is different from the scaling used for all other indicators in the model. These items ask for the percentage of the lesson time during one ordinary week (In a typical week of mathematics lessons for the TIMSS class, what percentage of time do students spend listening to lecture-style presentations? /BTBMPTLTS and In a typical week of mathematics lessons for the TIMSS class, what percentage of time do students spend taking tests or quizzes?/BTBMPTTQ). Teachers who take the responsibility for emphasizing and preparing the mathematics content in the instructional practise could also experience the time being limited for traditional lectures, tests and probations, and this could be because these teachers prioritise more time for other activities then traditional "teachers-desk instruction" The students on the other hand, at the same time answer that they often are listening to long briefings and having tests and probations, while they are not taking into account how large a part of the lesson-time these claim. Factor loadings for the indicators from the student-questionnaire unlike those from the teacher questionnaire thus are positive, but express the same conditions, namely teachers taking responsibility for emphasizing and preparing the mathematics content. These results suggest that it is possible to distinguish variation between individuals within and between classes concerning teacher responsibility for emphasizing and preparing the mathematics content.

For the latent factor SRL all factor loadings were found to be significant but somewhat higher. This latent factor is thus well indicated by the present subset of items concerning student’s opinion about
and opportunities for taking responsibility for their own learning process. The indicators BTBMASEA (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to explain their answers?) and BTBMASDL (In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to relate what they are learning in mathematics to their daily lives?) had weaker factor loadings than others. This could be interpreted as differences between classes due to these indicators are not so much appearance. Jointly for all indicators depicting this latent factor is that teacher responsibility for making student’s experiences and reasoning about the content visible is a common background factor.

Finally the latent factor CMC was found to be well captured by the current set of indicators with fairly high factor-loadings. This latent factor is mirroring to what extent students experience mathematical content with high relevance and conceptual orientation in the mathematics education. The observed variable BSBMHASM (In your math lessons, how often do you practice adding, subtracting, multiplying, and dividing without using a calculator?) indicates a non–routine work with arithmetic, BSBMHWFD (In your lessons, how often do you work on fractions and decimals?) indicates conscious work with such mathematical topics in which Sweden internationally have accomplished poor results. Finally, the observed variable BSBMHEFR (In your math lessons, how often do you write equations and functions to represent relationships?) is to be viewed as an indicator of conceptual rather than procedural mathematics, depending on the creative elements involved in creating models of mathematical relations instead of solving predetermined relations.

To sum up, the investigation of the correspondence between the model and data from TIMSS 2003 shows a satisfactory result. The model demonstrates the potential to describe the Swedish mathematics education among 8th graders along the lines of the dimensions in the model. This is however shown to be easier between students within the classes than between the classes. The substantial meaningfulness of the model and the possibilities of interpretations contribute to the assessment of model fit.

**DISCUSSION**

In this study a model is suggested for further analysing dimensions of instructional modes that supports second language students’ mathematical progress. The model challenges the traditional division into teacher- versus student-centred modes. Instructional modes sustaining both teacher responsibility for emphasizing and preparing the mathematics content and teacher responsibility for making second language student’s experiences and reasoning about the content visible in a way that enables interaction between teachers and students and where both have an active role in the learning process, are dimensions of such approaches. These, together with a third dimension concerning challenging mathematics content with high relevance and conceptual orientation, constitutes the theoretical base in a model for instructional modes developed in this study. For Swedish conditions, with the instructional practise “student’s independent work” where teacher responsibility for the
knowledge generation not can be taken for granted, it is of importance to expose this responsibility in a model for analysing dimensions of instructional modes.

By means of two-level confirmatory factor analysis, such a model has been developed from these starting-points. It has also been investigated how this model corresponds with data from TIMSS 2003, concerning Swedish 8th grade, and it is shown that the model has the potential to distinguish the Swedish mathematics education among 8th graders concerning the proposed dimensions.

The first dimension is represented by the latent factor labelled Teacher responsibility for content, but this is not equal to the traditional teacher-centred instructional mode, where the teacher mainly explains procedures and gives directions. It rather highlights the important aspect scaffolding, which indicates if the teacher is taking the responsibility for emphasizing and preparing the mathematics content in a way that helps second language students to reach new skills, new concepts and new levels of understanding (Vygotskij, 1978). It is of importance how second language students make use of their languages when learning mathematics (Cummins, 1984; Setati & Adler, 2000). When teacher e.g. give explicit explanations and rules for words and concepts the students are given a better chance to learn mathematics (Elmeroth, 2006; Jamieson, et al., 2004). The second dimension is labelled Student responsibility for learning, and it is indicated by observed variables dealing with opportunities for students to take responsibility for their own learning processes. The dimension mirrors if teachers do take the responsibility for making student’s experiences and reasoning about the content visible in a way that enables them to take this responsibility. From a Vygotskian holistic point of view (Vygotskij, 1978), the social action is to be viewed as a precondition for the individual action and the interpersonal and intrapersonal communication will affect the knowledge generation (Vygotskij, 1986). This dimension in the model could thus emphasize if teachers are taken the responsibility for arranging the instructional practise in order to enable this communication and thinking. The dimension is also indicating if students do relay to experiences and everyday life. However, the model does not has the potential to discern different qualities of context, which is a shortage. As was shown by Cooper and Harries (2002; 2005), mathematics education relayed to student’s previous experiences could treat second language students unfairly if the context used referred to other cultural discourses. This is a field for further studies and TIMSS data could be improved by adding observed variables concerning this phenomenon. Finally the dimension Challenging mathematics content is included in the model to make it possible to distinguish instructional practises depending on the existence of instruction encouraging the acquiring of previous knowledges and focusing on conceptual instead of mostly procedural mathematics. It is for second language learners crucial to use language in cognitive challenging situations, as Gibbons (2006) stated.

The correspondence of this model with data from TIMSS 2003 is good, which supports the possibility to adopt the presented alternative perspective on modes of instruction. In the first and the second latent factors, TRC and SRL, there are however on the Between-level some weak factor loadings. Differences between classes with respect to these indicators are thus not so apparent. It
had been desirable for these observed variables to indicate these factors in a more obvious way. One reason for why they do not seem to have this capacity could be validity problems with biased answers. The dominating opinion among teachers could influence to give “politically correct” answers. Teachers could e.g. be willing to answer “yes” to the question in BTBMASDL where it is inquired whether teachers ask students to relate what they learn to everyday life or not. The indicators thus lose in capacity to discern different instructional modes. There thus is room to improve the model.

An essential implication out of the results from this study is the potential of the developed model to expose important dimensions of teacher responsibility in instructional practice. Effects of the commonly occurring instructional mode “student’s independent work” in the Swedish mathematics education could further be examine, with a focus on effects for second language students. This could in a prolongation contribute to mathematics instruction more focused on teachers’ responsibility for knowledge generation. The present instructional mode, with main focus on students’ individual responsibility for their own learning, could treat second language students unfairly.
References


