Alternative Climate Policies and Intertemporal Emissions Leakage

Carolyn Fischer and Stephen Salant

Abstract

Efforts to limit cumulative emissions over the next century may be partially thwarted by the responses of fossil fuel suppliers. Current price-cost margins for major reserves are ample, leaving scope for significant price reductions if climate policies reduce demand for fossil fuels through conservation or substitution to clean alternatives. Most models simulating the consequences of climate policies completely disregard these supply responses. As for theoretical models, under standard assumptions they predict such strong supplier responses that climate policies may have no effect on cumulative emissions and may even leave society worse off, suffering damages from global warming sooner and with less time to adapt (the so-called “green paradox”). We contribute to this literature by developing a richer theoretical model that takes account of the different extraction costs and emissions rates of different fossil reserves. We use this model to compare the qualitative effects of four policy options: accelerating cost reductions in the clean backstop, taxing emissions, improving energy efficiency, and a clean fuel blend mandate; we also discuss the consequences of mandating carbon capture and sequestration. All policies can reduce cumulative emissions, but the backstop policy accelerates emissions while conservation policies (energy efficiency or blend mandates) delay emissions. We then calibrate the model using data on costs, reserves, and emissions/output ratios for five major categories of oil. Using this calibrated model, we estimate the intertemporal leakage rate—the percentage error in cumulative emissions reductions that would arise if no account is taken of the supply responses of oil producers. We find that conservation policies can have higher intertemporal leakage rates and backstop policies can have lower leakage than an emissions tax. Leakage rates generally decline as the policies become more stringent.

Key Words: green paradox, climate change, exhaustible resources

JEL Classification Numbers: Q3, Q4, Q5
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1. Introduction

Reducing emissions of the greenhouse gases (GHGs) that contribute to global climate change is the greatest collective action problem of our time. According to the Intergovernmental Panel on Climate Change (IPCC), stabilizing CO2 concentrations at levels that would avoid the largest risks of climate change could require global emissions to peak in the next 20 years (IPCC 2007). At the same time, the current United Nations Framework Convention on Climate Change (UNFCCC), under the principle of “common but differentiated responsibilities,” requires no mandatory action on the part of developing countries, including major emerging economies that are large emitters. Furthermore, in the absence of a binding successor to the Kyoto Protocol, not even developed countries are committed to emissions targets, although the Copenhagen Accord does call on countries to make individual pledges of action.

In this context of largely uncoordinated activities, several countries are taking significant steps to reduce their own GHG emissions. However, an important concern for unilateral movers is that their efforts may be partially (or completely) undermined by the actions of others.

Two channels of “carbon leakage” have been identified: spatial and intertemporal. With spatial leakage, the attempt by one government to raise the cost of fossil fuel use may drive economic activity toward unregulated, lower-cost countries. This type of leakage is likely to be small (a few percentage points) in terms of overall reductions.

We focus on the other channel of carbon leakage—the offsetting intertemporal responses of oil suppliers to a government’s attempts to curb fossil fuel usage. Current price-cost margins for some of the world’s largest reserves are ample, so there is scope for significant price

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reductions if clean substitutes eventually become cheap enough to threaten to lure consumers away from fossil fuels.\(^1\) Moreover since such fuels are in finite supply, current extraction decisions depend not only on current prices but also on future prices. If climate policies make selling fossil fuels in the distant future less attractive, suppliers may prefer instead to sell more in the present. Termined the “green paradox,” early studies of this phenomenon predicted that intertemporal leakage would reach 100%. We conclude that intertemporal leakage can be considerably smaller, particularly as policies get more ambitious.

Asked by *Foreign Policy*, “How can we stop climate change?,” Bjorn Lomborg (the “Skeptical Environmentalist”) replied “By being smart and investing in research to make green energy cheap instead of trying to make oil unaffordable” (*Foreign Policy* 2010). While such a policy might address some spatial leakage concerns, the prescription has received the greatest criticism in studies of the green paradox. Notably, Sinn (2008), who coined the term, argues that alternative energy strategies are particularly likely to accelerate rather than slow emissions over time. This acceleration can not only obviate any reductions in the long run but also increase the present discounted value of damages. In contrast, extraction taxes can at least be designed to slow fossil fuel consumption.\(^2\) But he argues more generally that policies to promote energy efficiency or to expand the use of clean substitutes are destined to speed global warming, while carbon sequestration is one of the few useful options for slowing it.

Other authors are more or less pessimistic about the prospects for clean energy policies. Strand (2007) makes a similar point about the indirect effects of reducing the cost of substitute technologies. Winter (2011) notes that with positive feedback effects between atmospheric carbon and the release of terrestrial carbon, innovation in clean energy technology can lead to a permanently higher temperature path. Grafton et al. (2010) find that subsidies to biofuels that are ongoing substitutes for fossil fuels may accelerate or delay extraction, depending on the relative

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1 A variety of studies using a *static* computable general equilibrium (CGE) models have shown the sensitivity of leakage to fossil fuel supply elasticities (e.g., Burniaux et al. 2009, Mattoo et al. 2009). Most of these studies find leakage rates in the range of 10-30% (Babiker and Rutherford 2005), representing both changes in fossil fuel markets and the shifting of other economic activities.

2 Hoel (2011) shows that emissions taxes that rise too quickly can accelerate emissions.
cost parameters. Chakravorty et al. (2010) show that greater potential for learning-by-doing in the substitute technology implies lower equilibrium energy prices, in order to deter innovation, resulting in increased resource extraction and greenhouse gas emissions. Other studies have combined the analysis of intertemporal and spatial emissions leakage. Hoel (2009) extends this analysis by assuming that countries differ in their taxation of fossil fuel use. Eichner and Pethig (2009) use a two-period model with separate abating, non-abating, and fossil fuel-supplying countries to explore the conditions under which tightening the emissions cap in the abating country accelerates global emissions.

In all of these models, the nonrenewable resource is ultimately exhausted, albeit at different rates; the cumulative carbon emissions are thus constant and the intertemporal leakage rate is 100%. Cumulative extraction is invariant in these models because of a combination of assumptions made about the extraction technology and backstop substitute, and by considering policies within a range that would not choke off fossil fuel demand.

In reality, extraction costs will rise as fossil fuels become increasingly scarce and reasonable climate policies can cause low-value resources to be left in the ground. In principle, this may be modeled either by positing a functional form for extraction costs that includes cumulative extraction as one argument or by assuming that different pools of oil have different per-unit costs. Gerlagh (2009) and Van der Ploeg and Withagen (2010) adopt the first approach. Gerlagh assumes that extraction costs are linear in cumulative extraction and finds that lowering the backstop cost decreases cumulative emissions but still increases initial emissions—an effect he terms a “weak green paradox.” Van der Ploeg and Withagen also assume that extraction costs increase with depletion and posit a range of costs for backstop technologies; they find that the green paradox still holds with cost reductions in expensive backstop technologies, but it need not arise with cost reductions in relatively cheap backstops.

In this paper, we adopt the second approach. We assume that oil pools differ in their per-unit cost of extraction, and hence extraction costs will rise over time as higher cost pools are accessed. This framework retains intuitive characteristics of the early studies and also allows us more flexibility when calibrating our model than if we had assumed a functional form of the cost
function. In addition, it allows us to take account of the different emissions/output ratios associated with the different types of fossil fuels, previously ignored in the literature.

We investigate the effects of four distinct climate policies:

1. accelerating the decline in costs of a carbon-free backstop technology,
2. taxing emissions,
3. improving energy efficiency, and
4. mandating a blend or portfolio ratio with the backstop technology.

Since the cost of implementing some of these (such as the first and third policy) is unknown, as is the damage resulting from a given path of cumulative emissions, assessing the welfare consequences of the different policies is impossible. Instead we require each policy to meet a given cumulative emissions target and compare the effects of the different policies on two summary measures: (1) the time interval before green technology replaces fossil fuels and (2) the degree of intertemporal leakage. The first metric relates to the weak green paradox; other things equal, policymakers may prefer longer time intervals to adjust to a given level of cumulative emissions. We show that, regardless of the number of pools assumed and their sizes and costs, the four policies can be ranked unambiguously in this dimension: for any given level of cumulative emissions, the backstop policy results in the least time to adapt, followed by the emissions tax, while the energy efficiency and blend mandate policies actually have identical effects and give society the longest time to adapt. The same rankings persist even if we assume that each conservation policy changes over time or that the emissions tax rises over time at the rate of interest.

For our second metric, we define the intertemporal leakage rate in a similar manner to the conventional spatial leakage rate: what is the change in emissions resulting from the rent adjustment as a share of the reductions that would occur in the absence of rent adjustment?
Quantifying the extent of intertemporal leakage is important since most climate policy models currently take no account of the dynamic responses of fossil fuel suppliers to policy changes.  

We thus shift focus from the time path of emissions, the emphasis of the prior green paradox literature, to the effectiveness in generating cumulative reductions. In a recent review of studies of the atmospheric lifetime of CO2, Archer et al. (2009) find a “strong consensus” across models of global carbon cycling that “the climate perturbations from fossil fuel–CO2 release extend hundreds of thousands of years into the future.” They further cite evidence that “the radiative impact of a kilogram of CO2 is nearly independent of whether that kilogram is released early or late in the fossil fuel era.” Given this longevity, according to the IPCC 4th Assessment Report, to reach a stabilization target of 450ppm would require cumulative emissions over the 21st century to be in the range of 1370 to 2200 GtCO2 (or 375 to 600 GtC; IPCC 2007). In comparison, Kharecha and Hansen (2008) estimated that there remain 70-140 GtC of natural gas, 120-250 GtC of conventional oil, 500-1,000 GtC of coal, and 150-1,000 GtC of unconventional oil. Especially if the upper range of reserve estimates hold, complete exhaustion of all proven resource pools, regardless of the time scale, would constitute a flagrant disregard for the GHG concentration targets.

Climate policy models suggest that significant policy changes will be required to achieve these targets. Estimates of intertemporal leakage will allow us to correct such forecasts by taking into account the supply responses that these models neglect. To gauge the magnitude of intertemporal leakage, we use a calibrated model of oil to compare the equilibrium effects under each policy to what would happen if scarcity rents did not adjust. In particular, we find that the

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3 For example, CGE models in studies like Burniaux et al. 2009 and Babiker and Rutherford 2005 focus attention on spatial leakage but ignore intertemporal leakage. Most integrated assessment models, including DICE, have non-scarce carbon fuels; exceptions are RICE-99 (Nordhaus and Boyer 2000), which only looks at carbon tax paths, and MERGE (Manne et al. 1995), which fixes production as a share of reserves and constrains the rate of new discoveries.

4 Stabilization of atmospheric concentrations would precede climate stabilization. According to the IPCC (AR4, 2007), “For most stabilisation levels global average temperature is approaching the equilibrium level over a few centuries.”

5 The policies may also affect cumulative emissions from other sources (e.g. coal). We confine attention here to the effect each policy has on cumulative emissions from the extraction and use of oil.
alternative energy policy is no more susceptible to intertemporal leakage than the emissions tax, although all policies suffer high rates of leakage at modest reduction targets.

The paper proceeds as follows. In Section 2, we describe a stylized one-pool model. In Section 3, we characterize the effects of the policies, each of which has been widely discussed as a way to reduce greenhouse gas emissions. In Section 4, we compare the consequences of these policies with respect to the time to transition to the backstop technology and generalize our analysis to \( n \) pools and time-varying policy paths. In Section 5, we use a calibrated version of the model to quantify intertemporal leakage rates associated with each policy. Section 6 discusses some limitations to conducting welfare analysis and Section 7 concludes the paper.

2. Single Pool Model

We begin by reviewing the behavior of the one-pool model in the absence of policy interventions. Suppose we have a single pool of oil of stock size \( (S) \) with a constant per-unit extraction cost \( (c) \), sold in a competitive market. A carbon-free backstop technology is available in unlimited capacity at constant marginal cost but it is initially too expensive to warrant consideration by consumers.\(^6\) Because of technological improvements, the (constant) marginal cost of this backstop, \( B(t; z) \), is assumed to decline exogenously over time toward a long-run cost \( B_{LR}(= \lim_{t \to \infty} B(t; z) < c) \).\(^7\) We assume that the parameter \( z \) can be increased by government policy. In the baseline scenario \( (z = z_0 > 0) \), we assume that this per-unit cost declines slowly enough that the pool of oil is completely exhausted before the backstop is utilized.

Let \( x_b \) denote the date when the backstop replaces oil. Denote the price consumers pay at time \( t \) as \( p(t) \); quantity demanded is \( D(p(t)) \).\(^8\) The interest rate is assumed exogenous and denoted

---

\(^6\) This assumption abstracts from several issues with transport fuel substitutes. Neither biofuels, hydrogen, nor electrification are necessarily carbon free; furthermore, marginal costs may increase with production levels at a large scale, such as due to land pressure.

\(^7\) In simulations, we will use the following functional form: \( B(t; z) = B_{LR} + (B_0 - B_{LR}) e^{-zt} \).

\(^8\) For clarity of exposition, in these sections we ignore any time trend in the demand function. However, in the parameterized numerical simulations, we will allow for demand growth.
as $r$. The aggregate flow of emissions at time $t$ (denoted $M(t)$) is assumed to be equal to the sum of quantity of oil produced at time $t$, multiplied by the emissions factor $\mu$: $M(t) = \mu q(t)$. We denote cumulative emissions as $E$. Therefore, $E = \int_0^{x_B} M(t)dt$.

The present value of a barrel of oil in the ground is represented by the shadow value ($\lambda$) of the cumulative extraction constraint. In a competitive equilibrium, the following conditions must hold: as long as the resource owner is extracting, the present discounted value of profit per unit must be constant (i.e., $p(t) = c + \lambda e^{rt}$ for $q(t) > 0$). If the backstop is in use, then the market price must equal the backstop cost (if $q_B(t) > 0$, then $p(t) = B(t,z)$). Thus, the path of the equilibrium price of a barrel of oil equivalent (BOE) is simply the smaller of two numbers: the price of oil and the production cost of the backstop (i.e., $p(t) = \min(c + \lambda e^{rt}, B(t,z))$). Since each argument of the min function is a continuous function of time, the price path is continuous. Finally, supply must equal demand at all points in time (so $q(t) = D(p(t))$ for $0 \leq t \leq x_B$, and $q_B(t) = D(p(t))$ for $t \geq x_B$).

We then have two potential regimes. In region (a), the pool of oil is fully exhausted (all of the oil eventually ends up “above ground”). In this case, the per-unit value of oil in the ground is strictly positive ($\lambda > 0$), and cumulative demand must equal the resource stock. Formally, we have two equations defining the two endogenous variables ($\lambda, x_B$):

$$\int_{t=0}^{x_B} D(p(t))dt = S \quad (1)$$

$$B(x_B; z) = c + \lambda e^{rx_B} \quad (2)$$

In region (b), the pool of oil is incompletely extracted (some oil remains “below ground”). In this case, the shadow value of oil must be zero ($\lambda = 0$), and the share extracted $\theta$ is determined by cumulative demand up to the switchover point. The following two equations define the two endogenous variables ($\theta, x_B$):
Figure 1 depicts the price path in the absence of a policy intervention (“no policy,” abbreviated “NP”), where \( z = z_0 \). All costs are expressed in terms of $ per barrel of oil equivalent (BOE).

**Figure 1: Price Path with No Policy**

3. Policies Intended to Reduce Greenhouse Gas Emissions

Next we introduce each of the four policies and analyze its effects in the simple one-pool model. To simplify, we assume that each policy is exogenously imposed\(^9\) and fully anticipated.\(^{10}\)

\(^9\)That is, we ignore the possibility that energy prices may themselves induce changes in energy efficiency or backstop R&D, or that CCS would be induced by the tax.

\(^{10}\)A fifth policy, mandating carbon capture and sequestration (CCS) has a different relationship between cumulative extraction and cumulative emissions and will be treated in the Appendix.
**Accelerating Backstop Cost Reductions**

Consider first a policy that accelerates cost reductions in backstop technologies after the first instant. That is, $B(0; z)$ is constant regardless of $z$, but $\frac{\partial B(t; z)}{\partial z} < 0$ for $t > 0$. Assume that $z$ increases above $z_0$ by so little that the pool would still be exhausted. An increase in $z$ will cause the backstop marginal cost to decline faster. If the price path did not change, the transition to the clean technology would occur sooner. But then the oil would not be exhausted, a disequilibrium. To restore equilibrium the scarcity rent must decline, and as a result, the entire price path falls, as Figure 2 depicts. Strengthening the policy thus lowers the rents on the resource and results in an earlier transition ($x_b$ falls). In this region the green paradox arises: faster reductions in the unit cost of the green technology do not reduce cumulative emissions (100% leakage) but the policy shortens the time until fossil fuels are exhausted and therefore raises the annual rate of emissions during the remainder of the fossil fuel era.

If the innovation rate ($z$) becomes large enough that the rents are driven to zero, we enter region (b). Oil would then sell at its marginal cost of extraction until the backstop enters. Faster innovation will not alter the price, but it does hasten the transition to the green technology (smaller $x_b$). It therefore increases the stock of reserves that remain in the ground rather than being transformed into greenhouse gases. Note that within this region (b), increasing $z$ does not alter the rate of extraction, so both the weak and strong versions of the green paradox disappear.

**Figure 2: Price Paths of the Two Regions with Backstop Policy**

\[
\begin{align*}
\text{NP} & \quad \text{Region (a)} \quad \text{Region (b)} \quad \text{pB} \\
\end{align*}
\]


**Emissions Tax**

An emissions tax levies a cost $\tau$ per unit of emissions at time $t$. For concreteness, we assume extractors pay the tax; however, the incidence would be the same if instead we had assumed that buyers pay the tax. For this single-pool model, since the emissions rate is invariant, the emissions tax is equivalent to an extraction tax.\(^\text{11}\)

Let $p(t)$ denote the price consumers pay. Extractors retain $p(t) - \tau \mu$ after paying the tax. In a competitive equilibrium, then, we have $p(t) = \min(c + \tau \mu + \lambda e^{\tau t}, B(t; z))$. While the cumulative extraction equations ((1) in region (a) and (3) in region (b)) remain the same (with the new expression for the price), the equations defining the switchover points for regions (a) and (b), respectively, must be modified:

\[
B(x_B; z_0) = c + \tau \mu + \lambda e^{\tau t}
\]

(2')

\[
B(x_B; z_0) = c + \tau \mu
\]

(4')

Figure 3 displays price paths within the different regions. If the emissions tax is sufficiently small, it will cause the price path to change but will still lead to complete exhaustion of each pool (region (a)). The new equilibrium price path must cross the old path from above. For, if the initial price on the new path were unchanged or smaller, the scarcity rent would have to be strictly smaller. But then the remainder of the path would lie strictly below the old path and cumulative demand would exceed the unchanged stock. So the initial price must be higher on the new path. To induce the same cumulative demand, the new path must cross the old one from above and the backstop will enter at a later date (larger $x_B$).

If the tax is just large enough to drive the scarcity rent to zero, the entire pool is still be exhausted ($\theta = 1$). The boundary of region (b) has been reached. The price consumers pay

---

\(^{11}\) Technically, we will assume that the tax rate is fixed relative to demand. In the five-pool section, we index the tax rate to demand growth, which ensures the tax effects are comparable over time. In the single-pool model without demand growth, the tax is fixed. We recognize that the form and path of taxes on fossil fuels influence market responses over time (e.g., Sinn 2008, Hoel 2011). An optimal emissions tax path would need to account for dynamics, damages, discounting, and the possibility of a time-inconsistency problem.
remains constant at \( c + \tau \mu \) until they switch to the backstop because it becomes cheaper. The higher the tax, the higher the price consumers pay for oil, the sooner they switch to the green backstop (smaller \( x_b \)), and the smaller is utilization of the pool (smaller \( \theta \)).

Figure 3: Price Paths of the Regions with Tax Policy

![Figure 3: Price Paths of the Regions with Tax Policy](image)

**Improvements in Energy Efficiency**

An alternative to promoting green substitutes or taxing emissions is to reduce the demand for oil by increasing the efficiency with which it is utilized. Examples include retrofit programs for buildings, energy efficiency standards for appliances or fuel economy standards for motor vehicles. Efficiency (denoted \( \phi \)) is measured in energy services per BOE. An improvement in this efficiency of utilization (an increase in \( \phi \)) has two countervailing effects: (1) it reduces the number of barrels required to obtain any level of energy services; but (2) it increases the level of energy services demanded by lowering their effective price—what has been termed the “rebound effect.” Whether an improvement in efficiency results in an increase in the demand for oil depends on the effective price elasticity (\( \eta \)) of the demand for services. If, for example, the increased efficiency raised services per barrel by 10%, but the decline in the effective price happened also to raise the demand for services by 10%, then there would be no change in barrels of oil demanded. If, however, the improved efficiency resulted in a smaller (larger) increase in the demand for services, the demand for oil would shift inwards (outward).
fact, the elasticity of the demand for services is small, so improvements in efficiency result in an inward shift in the demand for oil.

To clarify this discussion, we must distinguish between demand for energy services (denoted \( v \)) and the demand for oil (denoted \( q \)). Let \( \phi \equiv \frac{v}{q} \) denote services per barrel, assumed to be constant over time and subject to the influence of the policymaker. (Previously, we implicitly assumed that \( \phi = 1 \), so that there was no distinction between \( v \) and \( q \).) A consumer who values energy services maximizes \( U(v) - (p / \phi)v \) and purchases \( v \) units of services, where \( v \) implicitly solves \( U'(v) = (p / \phi) \).

If the oil price \( p \) remains constant but efficiency improves, then the effective price of services \( (p / \phi) \) falls and more of them will be consumed. To compute the derived demand for oil, invert the first-order condition to obtain: \( v = U^{-1}(p / \phi) \), or \( q = D(p / \phi) / \phi \), where \( D(x) = U^{-1}(x) \) is the demand for energy services (for policies that do not change energy efficiency, \( \phi = 1 \), and we simply have \( D(p) \)). Differentiating, we conclude that

\[
\frac{\partial D(p; \phi)}{\partial \phi} = \frac{d(p / \phi)[\eta(p / \phi) - 1]}{\phi^2},
\]

where \( \eta(x) \equiv -D'(x)x / D(x) \) is the elasticity of demand for energy services. Thus, an increase in efficiency cuts the demand for oil if and only if the magnitude of the elasticity of demand for services is smaller than 1. In that case, the rebound effect is dominated. Since the rebound effect is estimated to be smaller than 10\%, we assume that \( \eta < 1 \).\(^{12}\) Therefore improved efficiency causes the demand for oil to shift inward at any price.

To determine the effect of increased energy efficiency on the equilibrium price path, we must modify equations (1) and (3) representing cumulative extraction:

\[
\int_{t=0}^{\gamma} \frac{1}{\phi} D\left(\frac{c + \lambda e^r}{\phi}\right)dt = S
\]

\(1'\)

Equations (2) and (4) require no modification; as before, consumers switch from fossil fuels to the green backstop when the backstop becomes the cheaper energy source. Indeed, since the backstop price \( B(t; z_0) \) declines to the marginal cost of extraction \( c \) at the same date independent of the position of the demand curve, the switchover from fossil fuels to the clean backstop throughout region (b) always occurs at the same date \( x_B \).

Within region (a), improvements in EE decrease demand for oil and result in a price path that is uniformly lower. If the price path did not fall, the cumulative demand up until the switchover point would be less than the stock. Thus, to continue to exhaust the resource pool, the scarcity rent falls and the transition to the backstop occurs later. Emissions are postponed, but exhaustion still occurs.

If improvements in EE are sufficiently large, we reach the boundary with region (b), where the scarcity rent is just driven to zero, but nonetheless the entire high-cost pool is exhausted. However, further strengthening of EE policy has effects distinct from those of the previous policies. First, since the transition to the backstop occurs when the backstop price falls to the marginal cost of extraction \( (B(x_B; z_0)), x_B \) is unaffected by improvements in EE. Since improvements in EE reduce the rate of utilization of fuel without altering the date when it is replaced, they result in less cumulative usage of the oil stock (reduced \( \theta \)).

Figure 4 displays price paths for each of the regions. In fact, since the boundary conditions are independent of \( \varphi \), the diagram is the same regardless of the time path of the EE policy (although the quantity path would obviously differ).
Blend Mandate

A blend mandate would require that, for every unit of fuel supplied, a certain minimum percentage $\beta$ must come from the backstop substitute at time $t$, while the remainder can come from oil. This mandate is similar to a renewable fuel standard or renewable portfolio standard. The policy combines some of the effects of the emissions tax—paid in the form of a cost premium for the mandated share of energy from the backstop source—and some of the effects of the energy efficiency policy, since fossil fuels are being displaced in a given level of energy services with the backstop.

To sell its product, an extractor must blend one barrel of fossil fuel with $\beta / (1 - \beta)$ barrels of the backstop, and then sell the resulting $1 / (1 - \beta)$ barrels of the blended product at price $p_t$ per barrel of blend to obtain $p_t / (1 - \beta)$ per unit extracted. The extractor chooses the number of barrels to extract and blend each period to maximize:

$$\int_{t=0}^{\infty} \left( \frac{p_t}{1-\beta} - c - \frac{\beta}{1-\beta} B(t;z_0) \right) q_t e^{-\gamma} dt - \lambda \int_{t=0}^{\infty} q_t dt - S.$$  

So while the extractor is operating, the price must itself equal a weighted average of the two energy source costs: $p(t) = (1 - \beta)(c + \lambda e^{\gamma}) + \beta B(t;z_0)$. Meanwhile, at any given price,
only a fraction of the demand for barrels of blend is fulfilled by the fossil energy source: 
\[ q_t = (1 - \beta) D(p_t); \] the other \( \beta D(p_t) \) units are provided by the backstop component of the blend.

With the blend mandate, while the original equations (2) and (4) continue to govern the backstop switchover conditions for regions (a) and (b), respectively (with \( z = z_0 \)), equations (1) and (3) representing cumulative extraction in those regions must be modified:

\[ \int_{t=0}^{x_2} (1 - \beta) D\left( (1 - \beta)(c + \lambda e^{rt}) + \beta B(t; z_0) \right) dt = S \]  
(1'')

\[ \int_{t=0}^{x_2} (1 - \beta) D\left( (1 - \beta)c + \beta B(t; z_0) \right) dt = \theta S \]  
(3'')

Figure 5 displays the price paths of the two regions with the blend mandate. The mandate functions in part like a tax, raising costs and tilting the price path flatter as it becomes more stringent. However, when the backstop price declines over time more quickly than the blend mandate rises (as it does by definition with the fixed blend mandate), the implicit tax also declines over time, resulting in a declining price path in the more stringent policy regimes.

**Figure 5: Consumer Price Paths of the Regions with the Blend Mandate**

Larger blend requirements decrease demand for oil both by displacing oil and by raising the initial price. Within region (a), however, the price path cannot lie uniformly above the NP
path, else cumulative extraction would be less than the stock. Hence, the new price path must cross the no policy (or less stringent policy) path. Consequently, the switch to the backstop must occur later. As with the EE policy, emissions are postponed, but exhaustion still occurs.

With a sufficiently large blend mandate, we reach the boundary with region (b), where the scarcity rent is just driven to zero, but nonetheless the entire pool is exhausted. Region (b) is reached when the backstop marginal cost declines to the marginal extraction cost, exactly as in the case of the EE policy. From here, as with the EE policy, further strengthening of the blend mandate reduces cumulative emissions without affecting the date of transition to the backstop. In the case of the blend mandate, cumulative emissions fall (reduced $\theta$) because greater stringency reduces the amount of oil in the mixture and changes in the blend price encourage conservation.

4. Comparing Tradeoffs in Emissions and Backstop Transition

Transition Tradeoffs with a Single Pool

The time required to transition to the backstop while achieving a given cumulative emissions target is one indicator of the speed of emissions along the equilibrium path. With only one pool, reaching any cumulative extraction target below the baseline means operating in region (b). If we then focus on region (b), it is easy to use the equilibrium conditions to rank many of the policies in terms of the tradeoff between the transition time and cumulative emissions. This analysis also gives intuition in comparing policies in the $n$ pool case.

First, consider the effects of each policy on the backstop switchover time associated with a given level of cumulative extraction (and therefore emissions), $\theta S$. For simplicity, we assume that demand is time invariant, which allows for easy expressions of the stock equations (3).

For example, with the backstop policy, the improvement rate $z$ must be at a level such that, up until the switchover time, cumulative demand at the unit cost of oil equals the target level of extraction, or $x_B^{BS} = \theta S / D(c)$.

With a tax, then, we know that the consumer price of oil is higher than the unit cost, meaning that demand is lower in every time period up to the switchover point, which means that
to meet the same cumulative extraction, that switchover point must come later than with the backstop policy. I.e., \( x_{B_{\text{BS}}}^{\text{tax}} = \frac{\theta S}{D(c + \tau \mu)} > x_{B_{\text{BS}}} \), since \( \tau > 0 \).

The transition under the EE policy occurs when the backstop unit cost, without accelerated improvements, declines to the unit cost of oil. The transition under the blend policy occurs when the weighted average of (1) the unit cost of extraction and (2) the backstop unit cost—equals the backstop unit cost. But this can only occur if the two components to be weighted are equal to each other: \( B(x_B ; z_0) = (1 - \beta) c + \beta B(x_B ; z_0) \Rightarrow B(x_B ; z_0) = c \). Hence the \((E, x_B)\) tradeoff curves for the EE and blend-mandate coincide and this result is robust to alternative specifications.\(^{13}\) The stringency of either conservation policy does not affect the switchover point. The stringency simply determines the cumulative quantity of oil demanded until the switch occurs, which equals the extraction target.

Because the unit cost of oil is less than the cost inclusive of an emissions tax, the point where backstop and oil costs converge under conservation policies must be later than with the tax. (I.e., \( B(x_B^{\text{EE}} ; z_0) = c < c + \tau \mu = B(x_B^{\text{tax}} ; z_0) \)).

Thus, we have a complete ranking, for a given level of cumulative extraction: the timing of the transition to the backstop fuel is soonest for the backstop policy, then the tax policy, and last for the conservation policies, which have identical effects on that timing (i.e., \( x_B^{\text{blend}} > x_B^{\text{EE}} > x_B^{\text{tax}} > x_B^{\text{BS}} \)).

Figure 6 confirms this, by plotting cumulative emissions against \( x_B \) for each policy. We observe that in region (a), the backstop policy causes an earlier switch, while the tax delays it, and the EE/blend mandates delay even more. In region (b), the backstop and tax policies bring the switchover time forward monotonically, but they do not cross, while the EE and blend policies have no effect on the timing of the backstop transition.

\(^{13}\) The same considerations establish that in the \( n \) pool case, the transition from the marginal pool \( m \leq n \) to the backstop under the EE policy and under the blend policy occurs on the identical date. Indeed, the argument implies that even if the two policies are time-varying, their tradeoff curves coincide.
By this ranking of $x_B$, the “conservationist” policies (EE and the blend mandate) delay more consumption, while the backstop policy leads to the highest average emissions during the extraction period, given a cumulative emissions target.

**Extension to Multiple Pools**

With $n > 1$ pools, if the per-unit costs of extraction differ, they will be extracted in order of their extraction costs (Herfindahl, 1967). Moreover, in the equilibrium, a pool with a lower extraction cost will have a higher scarcity rent. Let $x_k$ denote the date of transition from pool $k-1$ to the pool $k$. The set of equations defining these endogenous variables is described in Section A1 of the Appendix.

Suppose we gradually tighten one of the policies considered previously. Then the equilibrium will fall successively into each of $2n$ qualitative regions $R_{ma}, R_{mb}, \ldots, R_{ia}, R_{ib}$. In any of the region (a)’s, every pool that is utilized will ultimately be exhausted and the associated scarcity rents will each be strictly positive. Strengthening the policy within a region (a) causes the rents to decline until the lowest rent reaches zero. Further strengthening of the policy moves the equilibrium into a region (b). In such regions, the last pool utilized has a zero scarcity rent and will (except for the boundary case) be only partially exhausted. Further strengthening of the
policy within region (b) crowds out extraction of the marginal pool; scarcity rents for the inframarginal pools can be further eroded by policies that influence demand. When the resource pool with the zero rent ceases to be utilized at all \((\theta_m = 0)\), the equilibrium falls into the next (a) region, and so on.

The pattern of tradeoffs between cumulative emissions and the timing of the transition to the backstop—henceforth the “transition tradeoff curve”—also follows that of the single pool, albeit now in a zig-zag pattern following the regions. In each region (a), cumulative emissions are flat (indeed, at the level of \(\sum_{i=1}^{n} \mu_i S_i\) for the marginal pool \(m \leq n\)), and policy stringency only affects the backstop switchover \(x_B\). Within region \(R_{m,a}\), accelerating backstop cost reductions results in a uniformly lower price path and hence, to maintain cumulative demand equal to the unchanged sum of the stocks, the backstop must replace fossil fuels earlier \(x_B\) declines). If instead we use an emissions tax, increasing the tax rate in that region reduces consumption early on and causes the price path to tilt (by the same logic as the one-pool model), so reaching the same cumulative consumption requires switching to the backstop later \(x_B\) increases). With either the energy efficiency policy or the blend mandate, as the policy is gradually strengthened within region \(R_{m,a}\), demand for fossil fuels falls in every time period, so the same cumulative extraction must be met with a later transition to the backstop \(x_B\) increases).

The transition tradeoffs change from the (a) to the (b) regions, in similar manner to the single pool. For any \(mth\) pool as the marginal one, once region \(R_{m,b}\) is reached, accelerating technical change further means the backstop cost falls to the unchanged per unit cost \(c_m\) of the marginal pool sooner; with demand being unaffected by the backstop policy, the timing of the switch to the \(mth\) pool is unchanged, so cumulative extraction from the marginal pool declines as well as the switchover timing \(x_B\) decreases). Under the emissions tax, a tax increase raises the otherwise unchanged cost of the marginal pool \(c_m + \tau \mu_m\), which the backstop requires less time to meet \(x_B\) decreases). The tax also raises costs and erodes scarcity values for the inframarginal pools, tilting the price path and delaying the end date of extracting the preceding pool \(x_{m-1}\).
when extraction from the marginal pool commences.\footnote{Let $x_m$ denote the date when extraction from pool $m-1$ ceases and extraction from pool $m$ commences. Then $x_m$ solves the following equation: $c_{m-1} + \mu_{m-1} + \lambda_{m-1}e^{x_m} = c_m + \mu_m$. This can be rewritten as: $e^{x_m} = (c_m - c_{m-1} + (\mu_m - \mu_{m-1})\tau) / \lambda_{m-1}$. Since the tax parameter in the numerator is strictly positive (and recall we are maintaining the assumption that $\mu_m \geq \mu_{m-1}$), an increase in the tax rate raises the numerator, lowers the shadow value that is the denominator, and raises the quotient. As a result, $x_m$ increases monotonically in $\tau$.} Therefore, incremental tax increases in this region shorten the extraction time of the marginal pool and reduce cumulative emissions.

The transition tradeoff curves under the energy efficiency policy and the blend mandate policy coincide again, as increases in stringency leave the backstop switchover timing unchanged in region $R_{m,b}$.

Remarkably, across all the regions, these three transition tradeoff curves touch but never cross, permitting us to rank the policies. It is easy to show that the changeover from $R_{m,a}$ to $R_{m,b}$ occurs sooner under the emissions tax than under the energy efficiency or blend mandate: Under the emissions tax, the backstop marginal cost must only decline to $c_m + \mu_m$; more time is required for it to decline to $c_m$, as with the conservation policies.

It remains to show that the transition tradeoff curve under the emissions tax never crosses that of the backstop policy. Since the curves go in opposite directions in the flat (a) regions, such a crossing would have to happen in a (b) region. Suppose the two curves did cross in region $R_{m,b}$ for some $m = 1, \ldots, n$ so that both policies generated the same cumulative emissions and induced backstop entry at the same date. The marginal pool $m$ would have to sell for a higher price ($c_m + \mu_m$) under the tax policy than under the backstop policy (where the price is $c_m$). Since emissions from both terminate, by assumption, at the same instant ($x_y$), the only way that cumulative emissions could be as large under the tax is if extraction from the marginal pool begins sooner than it does under the backstop policy; however, we have shown (in Footnote 14) that in $R_{m,b}$ an increase in the tax rate delays the start point of extraction from the marginal pool.
Hence, if both policies induce an equilibrium with the same marginal pool partially utilized and replaced at the same instant by the backstop, the emissions tax must generate smaller cumulative emissions than the backstop policy. Consequently, the two tradeoff curves cannot cross, and we retain the same relative ranking as with the single pool.

**Extension to Time-Varying Policies**

We have seen that the ranking of the transition tradeoff curves of the four policies remains the same regardless of the number of pools available to the extractors. In preparation for our simulations, we note that the ranking also remains unchanged for the time-varying policies we will consider. The backstop policy was already assumed to be time-varying. Suppose the EE policy varies over time so that the demand for oil is non-stationary ($\phi$ is weakly increasing and a monotonic function over time). It remains true that in every region (a) some pool ($m$) will be the highest cost pool to be utilized (the “marginal pool”) and the transition tradeoff curve will have a horizontal segment at a height equal to $\sum_{i=1}^{m} \mu S_i$ exactly as in the case when the EE policy was stationary. Moreover, every region (b) still occurs at the same date as before—when the backstop marginal cost descends to the marginal cost of the marginal pool: $B(x_B; z_0) = c_m$. Hence, the relationship between cumulative emissions and $B(x_B)$ is unaffected even if the stringency of the EE policy varies over time. The same logic can be applied to a time-varying blend mandate.

It remains to discuss the effects of introducing nonstationarities in the emissions tax. As tax paths have received prior attention, we confine our attention to the case in which the initial emissions tax is set by the policymaker at $\tau$ and is then raised at the rate of interest so that $\tau(t) = \tau e^{rt}$. This path corresponds to the optimal tax for meeting a cumulative emissions constraint. If the policy is stringent enough that $m$ is the marginal pool, then the induced price path is continuous, consisting of $m$ strictly convex regions separated by kinks (with the left

15 As Sinn (2008) and Hoel (2011) point out, the time path of emissions fees or extraction taxes matters for the present discounted value of emissions, given a cumulative emissions outcome.
derivative strictly larger than the right derivative). During region (a), when exhaustion of the $m$th pool remains complete, the shadow values simply absorb the tax cost of the last pool, and $x_b$ remains unchanged; the switchover is neither delayed nor accelerated.\footnote{As shown in Appendix A2, if pool $m$ is marginal in region (a) then the equilibrium price path is $p(t) = \min(c_m + [\mu_m + \lambda_m]e^\tau, ..., c_1 + [\mu_1 + \lambda_1]e^\tau, B(t; z))$. Suppose first that the tax is zero. Then in the equilibrium $\lambda_1 > \lambda_2 > ... > \lambda_m$. If $\tau$ is marginally higher, the individual multipliers will decrease so that each term in square brackets remains unchanged; since lower cost pools have higher multipliers but are assumed to have lower emissions to output ratios, larger multipliers must decrease by less than smaller multipliers and the order of the multipliers is preserved. It is straightforward to verify that this hypothesized new assignment of multipliers generates a competitive equilibrium: since the magnitude of the terms in each square bracket is unchanged, the price path would be unchanged and cumulative demand for the reserves in each pool would still equal the stock in each pool. Moreover, as asserted in the text, the switchover to the backstop would occur on the same date.} Once $\tau$ rises enough such that $\lambda_m = 0$, then $p(t) = c_m + \mu_m e^\tau$ and the marginal pool is partially depleted while lower cost pools are exhausted. At this point in region (b), an increase in $\tau$ shifts the price path faced by consumers upward uniformly. Since at higher prices it takes longer to exhaust each inframarginal pool, each transition to the next pool occurs later and cumulative depletion from the marginal pool is smaller. When the tax is sufficiently high, that pool is never utilized and the next most costly pool becomes the marginal pool.

Since the transition tradeoff curve that results as $\tau$ varies has no horizontal segments in region (a), $x_b$ decreases monotonically as $E$ falls. Since the transition tradeoff curve common to the time-varying EE and blend policy is nondecreasing as $E$ falls, that curve cannot intersect the tradeoff curve for the backstop policy except at the point representing no policy. Finally, as in the non-time-varying case, the transition tradeoff curve for the time-varying emissions tax also has no nontrivial intersection with that of the backstop policy. For any common transition time with positive policy levels, cumulative demand and therefore emissions must be smaller with the time-varying emissions tax than the backstop policy. Hence, these two tradeoff curves cannot cross and the ranking of the four policies remains unchanged even if they vary over time as described above.
5. Comparing Intertemporal Leakage Rates

The fundamental problem of the green paradox is the acceleration of consumption that arises from falling scarcity rents. The transition tradeoff curves compare average annual emissions during the period of exploitation, a measure of the speed of emissions along the path. While this measure takes into account the effects of scarcity rent equilibration, it also includes policy-induced changes in the emissions path that would occur in the absence of rent adjustments.

An alternative perspective is to focus on the degree of intertemporal leakage—that is, the magnitude of the additional emissions reductions under each policy that would arise if rents did not re-equilibrate but instead remained fixed at their no policy levels. This alternative perspective has practical value since one can adjust the forecasts of the predicted reduction in cumulative emissions made by models that do not account for adjustments in scarcity rents. To assess the potential magnitudes of intertemporal leakage, we simulate the effects of each policy using a model calibrated to reflect real-world data. All policies suffer from intertemporal leakage, and the rankings can be quite different from the transition tradeoffs.

Parameterized Five-Pool Model

We draw on the literature to parameterize a multiple pool model reflecting the five major types of oil: Middle East and North African (MENA) conventional oil, other conventional oil, enhanced oil recovery (EOR) and deep-water drilling, heavy oil bitumen (including oil sands), and oil shales. This level of disaggregation is sufficient to capture the effects. For each pool, we specify the size, per-unit cost, and emissions factor. On the demand side, we draw on empirical estimates of demand elasticity and projections of demand growth over time.

Estimates of oil reserves and costs vary widely. EIA currently estimates global proven reserves to be about 1200 billion barrels (including conventional and some unconventional like Canadian oil sands). Kharecha and Hansen (2008) report reserves estimates in GtC, but if converted to billion barrels of oil equivalent (BBOE), they find a range of 1000-2100 BBOE of conventional oil and 1300-8500 BBOE of unconventional oil. Aguilera et al. (2009) include projections of future reserve growth, leading to estimates of conventional oil reserves of 6000-
7000 billion barrels available at prices as low as $5/barrel, heavy oil reserves of 4000 billion barrels at $15/BOE, oil sands reserves of 5000 billion barrels at $25/BOE, and up to 14,000 billion barrels of oil shale that could be tapped at $35/BOE. For our purposes, we draw rough estimates from the fall 2010 International Energy Agency (IEA) report, which gives a range of production costs and available reserves by oil type (Figure 7).

Our specific reserves and cost assumptions are given in Table 1. To convert to CO2 emissions (right column), we assume (as suggested by U.S. EPA) that a barrel of oil contributes 0.43 tons\(^{17}\) of CO2 and adjust for the fact that different unconventional sources have larger emissions factors relative to conventional oil.\(^{18}\)

Figure 7: (Source: IEA 2010)

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\(^{17}\) [http://www.epa.gov/grnpower/pubs/calcmeth.htm](http://www.epa.gov/grnpower/pubs/calcmeth.htm)

\(^{18}\) See Table 3-2 of the California technical analysis of the low-carbon fuel standard [http://www.energy.ca.gov/low_carbon_fuel_standard/UC_LCFS_study_Part_1-FINAL.pdf](http://www.energy.ca.gov/low_carbon_fuel_standard/UC_LCFS_study_Part_1-FINAL.pdf)
Table 1: Reserves and Cost Assumptions

<table>
<thead>
<tr>
<th>Oil reserve source</th>
<th>Available Reserves (BBOE)</th>
<th>Production cost</th>
<th>Relative Emissions Factor</th>
<th>CO2 (Gt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle East/N. Africa conventional</td>
<td>900</td>
<td>$17</td>
<td>1</td>
<td>387</td>
</tr>
<tr>
<td>Other conventional</td>
<td>940</td>
<td>$25</td>
<td>1</td>
<td>404</td>
</tr>
<tr>
<td>EOR and deep water</td>
<td>740</td>
<td>$50</td>
<td>1.105</td>
<td>352</td>
</tr>
<tr>
<td>Heavy Oil/Oil Sands</td>
<td>1780</td>
<td>$60</td>
<td>1.27</td>
<td>972</td>
</tr>
<tr>
<td>Oil Shale</td>
<td>880</td>
<td>$70</td>
<td>2</td>
<td>757</td>
</tr>
<tr>
<td>Biofuels / backstop technology</td>
<td>Unlimited</td>
<td>$100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The assumed initial backstop marginal cost is drawn from a range of common estimates of biofuels, in line with the IEA estimates; although conventional biofuels like sugarcane ethanol are currently cheaper, the second-generation fuels like cellulosic ethanol and biodiesel—which have greater potential for larger scale supplies needed to function as backstop technologies—have higher costs.\(^{19}\) For this exercise, we assume that backstop costs start at $100 and will ultimately asymptote to $10 (i.e., be lower than conventional oil in the far future), following a modest no-intervention cost reduction rate of 0.25% per year of the excess over the long-run cost \((z = 0.0025)\). The combination of these cost assumptions ensures that all oil resources would be fully exhausted by the end of the century in the absence of policy interventions. We assume that the backstop fuels are non-emitting.\(^{20}\) While we draw on biofuels in making these cost estimates, we recognize that future backstops could also include other options like hydrogen or clean electricity for plug-in vehicles.\(^{21}\)

For the demand side of the simulation model, we parameterize a linear demand function. According to the Energy Information Administration (EIA), global annual oil consumption has been roughly 86 million barrels per day in recent years, or an annual consumption of 31.4 billion barrels.\(^{22}\)

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\(^{19}\) In 2007, USDA estimated cellulosic ethanol production costs at $2.65 per gallon, compared with $1.65 for corn-based ethanol.

\(^{20}\) We acknowledge the actual emissions factors for biofuels, particularly those associated with land use changes, are controversial.

\(^{21}\) Of course, synthetic fuels derived from coal or natural gas could also be substitutes, but we assume fossil-based backstops are precluded.

\(^{22}\) [http://tonto.eia.doe.gov/cfapps/ipdbproject/IEDIndex3.cfm?tid=5&pid=54&aid=2](http://tonto.eia.doe.gov/cfapps/ipdbproject/IEDIndex3.cfm?tid=5&pid=54&aid=2)
We assume an effective elasticity of -0.25. This value roughly corresponds to the median estimate of a global oil demand elasticity from Killian and Murphy (2010). Earlier estimates of the price elasticity of demand for gasoline (primarily in the U.S.) find short-term demand elasticities of about -0.25 and long-run elasticities of about -0.6 (Espey (1996) and Goodwin et al. 2004). On the other hand, Cooper (2003) and Dargay and Gately (2010) find much lower price elasticities of demand (-0.15 and smaller) when considering a broader array of countries, particularly non-OECD countries, and more recent time periods. However, Killian and Murphy (2010) warn that most studies of such elasticities using dynamic models have been econometrically flawed by not accounting for price endogeneity.

EIA’s International Energy Outlook 2010 projects global demand to increase 49% from 2007 to 2035, or about 1.45% per year, primarily from developing countries. We incorporate demand growth by assuming that the linear demand curve shifts out at this rate without changing slope. We position the initial demand curve so that (1) it passes through the quantity 31.4 BBOE, (2) it has a point elasticity at that quantity of -.25, and (3) the initial price on the equilibrium price path in the base (no policy) case induces a quantity demanded of 31.4 BBOE. Solving, we find that initial price to be $41 per barrel.

Of course, our simple Hotelling model does not explain the simultaneous exploitation of high-cost resources alongside low-cost ones and predicts an initial price of $41/barrel instead of $75 per barrel. However, our modest additions do lend a great deal more realism to a model that still allows for the kinds of green paradoxes explored in the literature.

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24 Gaudet, Moreaux, and Salant (2001) show how to generalize the Hotelling model to the case where the location of demanders (as well as reserve deposits) is exogenously distributed. In such a model, resources pools are sometimes accessed simultaneously by spatially distributed users even though the pools differ in extraction costs (a high-cost pool might be located near one consumer and a lower-cost pool might be located near another consumer, with the two pools far apart from each other). Despite its greater realism, we declined to use this spatial Hotelling model in our preliminary investigation. We decided to use the nonspatial Hotelling model instead since that model has been used by all of the other contributors to the Green Paradox literature. Our goal here is to clarify how the introduction of heterogeneity in the extraction costs and emissions factors of the different pools of fossil fuels alters conclusions others have drawn about the magnitude of intertemporal emissions leakage.
Figure 8 displays the no-policy price path indicated by the five-pool model. We see that differentiating among more pools leads to a smoother price path. Demand growth outpaces price growth, so corresponding consumption rises smoothly over time, and fossil fuels are exhausted after 83 years.

**Figure 8: No Policy Price Path with Five Pools**

![Graph showing price path with five pools](image)

**Simulated Transition Tradeoff Curves under the Four Policies**

Figure 9 displays the relationship between cumulative emissions and the length of time to switch to the backstop for the five-pool model. The emissions tax is time varying, rising at the interest rate. As previously discussed, the transition tradeoff curves for the conservation policies are independent of the policy growth path.

With the greater number of pools, we notice that the regions in which the marginal pool is fully extracted are less pronounced than in the one-pool model, leading to a smoother relationship between the switchover timing and cumulative emissions. The difference between the backstop and emissions tax policy is also smaller. Hence, average emissions under the two

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25 We also simulated a tax that rises at the rate of demand growth but the difference was insignificant. For a given cumulative emissions, the slower growth tax path delays the switchover by less than 2%. We did not consider a fixed tax, since with the parameterized demand growth, the tax levels required to meet reduction targets would choke off demand in the early years.
policies are similar. The energy efficiency and blend mandates still greatly delay the arrival of the backstop.

Figure 9: Simulated Tradeoff Curves for the Tax and Emissions Policies (left panel) and the Conservation Policies (right panel)

**Simulated Leakage Rates Given a Cumulative Emissions Target**

Next we measure each policy’s susceptibility to intertemporal leakage. In the absence of rent adjustment, policy $i$ would induce emissions reductions of $E_{NP} - E_{iNL}$. Leakage under policy $i$ is defined as the extra emissions that occur after rents adjust compared to their level in the absence of a rent adjustment: $E_i - E_{iNL}$. The leakage rate under policy $i$ ($L_i$) is the leakage as a percentage of the emissions reduction that the policy would induce in the absence of rent adjustment: $L_i = (E_i - E_{iNL}) / (E_{NP} - E_{iNL})$. Equivalently, $(E_{NP} - E_i) = (E_{NP} - E_{iNL})(1 - L_i)$. The right-hand side is the product of two factors: (1) the reduction in emissions in response to policy $i$ predicted by a model that takes no account of the induced change in scarcity rents and (2) the complement of the simulated leakage rate. The left-hand side is the reduction in cumulative emissions under policy $i$ predicted to occur after scarcity rents adjust. Thus if a model that takes no account of the change in scarcity rents predicts that policy $i$ will cut emissions by 20 tons and

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26 This definition is intended to mimic the common definition of spatial emissions leakage.
the simulated leakage rate for that policy is 60%, then after scarcity rents equilibrate we predict the policy will cut emissions by only 8 tons (40% of 20).

Figure 10 illustrates the extent of intertemporal leakage for the case of the (optimal) emissions tax in the five-pool model. The solid line shows the prediction of the actual cumulative emissions reductions induced by the policy after rents re-equilibrate: \( E^{NP} - E_i \). The dashed line indicates the consequences of the policy if rents remained fixed at the no policy level (no intertemporal leakage): \( E^{NP} - E^{NL}_i \). Leakage is the vertical difference between the lines: \( E_i - E^{NL}_i \). The leakage rate divides that amount by the height of the dashed line. Leakage increases within the (a) regions and decreases during the (b) regions.

Figure 10: Emissions Reductions as a Function of Policy Stringency with (Solid Line) and without (Dashed Line) Rent Adjustment

From these simulations, we can calculate the average intertemporal leakage rates associated with a given level of cumulative emissions. We do this for a range of time-varying and stationary policies. Although policy paths have little influence on the backstop transition tradeoffs, they can have a large influence on the leakage tradeoffs, particularly for the conservation policies. Figures 11 and 12 depict these leakage tradeoff curves. The horizontal segments in the diagram arise because an interval of stringencies inducing the same cumulative emissions will induce different leakage rates. Policies to the left have less leakage, on average.
We see that all policies initially have 100 percent leakage, and that rate declines as cumulative emissions fall.

Figure 11 compares the backstop policy to an emissions tax that rises at the rate of interest and an emissions tax that rises more slowly at the rate of demand growth. The emissions tax policies have less leakage than the backstop policy initially, in part due to their ability to differentiate among higher emissions intensity pools. However, for more dramatic reductions, the backstop has lower leakage rates than the tax policies. Meanwhile, the slower tax path that is associated with somewhat more delay in the backstop transition has a consistently (though not greatly) higher leakage rate than the emissions tax rising at the rate of interest. Particularly after the extraction of the highest cost/highest emitting pool is eliminated, the leakage rate differences among all three fall within 5 percentage points of each other.

Figure 11: Leakage Tradeoff Curves for Backstop and Tax Policies

Figure 12 compares the leakage tradeoff curves for two variants of each conservation policy, as well as the backstop policy as a reference. With “EE Fixed” and “Blend Fixed” we simulate the policies as described in the one-pool model: the mandates require an immediate and permanent improvement in EE (or similarly a blend ratio). These policies have nearly identical
effects and are associated with consistently higher leakage rates than all other policies. With “EE Growing” and “Blend Growing”, we assume that the mandates require an annual rate of improvement in efficiency or the backstop blend. We find that delay in raising the stringency of the conservation policies improves their performance with respect to intertemporal leakage. Although their leakage rates are still higher at more modest targets, they outperform the backstop and emissions tax at more ambitious reduction targets.

Figure 12: Leakage Tradeoff Curves for Conservation Policies, Compared to the Backstop Policy

Sensitivity

The qualitative results are similar when we define leakage rates in terms of the present value of emissions, which has received some attention in the prior green paradox literature. The relative backstop and emission tax leakage rates change little, but the conservation policies

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27 Specifically, $\beta_t = 1 - e^{-bt}$ and $\phi_t = e^{bt}$.

28 The present value leakage rate is defined analogously to the cumulative emissions intertemporal leakage rate: $PVL_t = (PVE_t - PVE^{\infty}_t) / (PVE^{\infty} - PVE^{\infty})$, where $PVE = \int M(t) e^{-\beta t} dt$. This metric is useful if one is concerned about the time path of emissions damages and believes that damages at any date are roughly a linear function of cumulative prior emissions. Cumulative emissions are preferable if one is concerned about potential threshold effects and long lags in climatic response that require setting a cumulative emissions budget for the time period.
do perform better. In particular, the fixed mandates have the lowest present value leakage rates when the policies are relatively modest and only crowd out oil shale.

We also perform sensitivity analysis with respect to some key parameters in the numerical simulations. First, we lower the elasticity of demand for fuel from 0.25 to 0.1, closer to short-run elasticity estimates. The initial switchover times are slightly delayed, and the difference between the backstop and tax policies in the transition tradeoff curves are more compressed. However, the leakage rate tradeoff curves are indistinguishable from the higher elasticity baseline. Second, we vary our assumptions about resource scarcity. If all reserves prove to be fifty percent larger than in the baseline, initial prices would be lower, transition horizons longer, and leakage rates smaller for modest targets, although the qualitative results are unchanged.

Finally, carbon capture and sequestration (CCS) has been proposed as one of the few viable options for addressing the green paradox, as it allows for reductions to occur while oil continues to be extracted and consumed (Sinn 2008). As such, it deserves some separate discussion, and we treat it formally in the Appendix. Essentially, a mandate requiring the equivalent of a certain share of emissions to be captured and sequestered functions like an implicit emissions tax. Thus, it has the same effect on the extraction path and price path as a corresponding carbon tax. However, since the tax revenues are used to buy sequestration, a given amount of extraction is associated with fewer emissions. The results indicate that while CCS does induce more emissions reductions than the equivalent carbon tax, its susceptibility to intertemporal leakage is not that different in magnitude.

6. Limitations for Welfare Analysis

Since reliable estimates of the cost of accelerating cost reductions in alternative fuels and the cost of permanently improving in energy efficiency do not exist, we can conduct neither a meaningful welfare nor cost effectiveness analysis. However, making a rough estimate of the policies needed to meet mitigation goals suggests the magnitudes of the potential costs.

Table 2 compares the levels of policy stringency required to achieve given levels of extraction in the simulation model. For example, to avoid the emissions of the oil sands and shale
reserves requires a $17/ton CO2 tax (within the range of the EU ETS allowance prices over the past year), or an increase in the rate of cost reductions in cellulosic biofuels by 1% per year, a 2.9% annual improvement in energy efficiency, or a 2.7% annual reduction in the share of fossil sources in the fuel blend.

Table 2: Levels of Policy Stringency Required

<table>
<thead>
<tr>
<th>Region</th>
<th>Backstop (Annual Improvement Rate)</th>
<th>Present Value of Emissions Tax</th>
<th>Energy Efficiency (Annual Improvement Rate)</th>
<th>Blend Mandate (Annual Reduction in Fossil Share)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Policy</td>
<td>0.25%</td>
<td>$-</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>No Shale Rents, Full Extraction</td>
<td>0.5%</td>
<td>$2.93</td>
<td>1.6%</td>
<td>1.5%</td>
</tr>
<tr>
<td>No Shale Rents or Extraction</td>
<td>0.6%</td>
<td>$3.83</td>
<td>1.9%</td>
<td>1.8%</td>
</tr>
<tr>
<td>No Oil Sands Rents, Full Extraction</td>
<td>0.8%</td>
<td>$10.14</td>
<td>2.1%</td>
<td>2.1%</td>
</tr>
<tr>
<td>No Oil Sands Rents or Extraction</td>
<td>1.1%</td>
<td>$17.49</td>
<td>2.9%</td>
<td>2.7%</td>
</tr>
<tr>
<td>No Deepwater Rents, Full Extraction</td>
<td>1.6%</td>
<td>$27.15</td>
<td>2.9%</td>
<td>2.7%</td>
</tr>
<tr>
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<td>$35.54</td>
<td>3.5%</td>
<td>3.3%</td>
</tr>
<tr>
<td>No Conventional Rents, Full Extraction</td>
<td>4.7%</td>
<td>$63.43</td>
<td>3.5%</td>
<td>3.3%</td>
</tr>
<tr>
<td>No Conventional Rents or Extraction</td>
<td>8.3%</td>
<td>$93.72</td>
<td>5.7%</td>
<td>5.0%</td>
</tr>
<tr>
<td>No MENA Rents, Full Extraction</td>
<td>12.1%</td>
<td>$104.57</td>
<td>5.7%</td>
<td>5.0%</td>
</tr>
<tr>
<td>No MENA Rents or Extraction</td>
<td>infinite</td>
<td>$193.02</td>
<td>infinite</td>
<td>infinite</td>
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</tbody>
</table>

Even with reliable cost estimates, additional limitations prevent us from performing a proper welfare analysis. In particular, there is considerable uncertainty about the shape of the damage function.

7. Conclusion

Climate change is a long-term problem, and since GHGs decay quite slowly, stabilizing their atmospheric concentrations requires something akin to a limit on cumulative emissions over the next century. Concern over the green paradox takes two main forms. One is that efforts to reduce GHG emissions may be undone in part or in whole by emissions leakage, not only across countries but over time, given that the major sources of GHGs are exhaustible resources. A more subtle form of the green paradox is that, not only may emissions leak over time, but
some efforts to spur a transition to clean energy may accelerate emissions in such a way that the present value of the damages of climate change may actually increase. However, for understanding the true extent of the green paradox, we find it is important to isolate the effects of intertemporal leakage—that is, the change in emissions that can be attributed to market adjustments in scarcity rents—from the effects of the policies themselves on the timing of emissions.

Our study reinforces earlier findings that accelerating cost reductions in a clean backstop technology tends also to accelerate extraction of nonrenewable resources; however, earlier transition to a backstop can reduce cumulative emissions when exhaustion will not be complete. Furthermore, in comparing the intertemporal leakage rates for cumulative emissions, we find the backstop policy can actually outperform other policy alternatives, including an emissions tax. Although an emissions tax can slow emissions in a context of complete exhaustion, when applied at levels to reduce cumulative extraction, it also accelerates the arrival of the backstop. Meanwhile, energy efficiency improvements and clean energy blend mandates delay emissions and the adoption of the backstop technology (at least, they never accelerate that adoption). But if the greater concern is cumulative emissions, they also suffer from considerable leakage rates, particularly policies that take stronger conservation action early on. While all policies considered can reduce cumulative emissions under the right circumstances, the CCS mandate was the only policy that did it in all circumstances. Yet, that does not imply it has less of a problem with intertemporal leakage than other policies. For all of the policies, leakage rates are highest when policies are weak, but as reduction targets become more stringent, leakage rates tend to fall.

We have noted some important simplifications in our analysis. Even with the given policies, additional assumptions would be needed to address questions of the relative cost effectiveness of meeting a given cumulative emissions target. We have not explicitly represented the costs of EE improvements, backstop technology policy, or CCS. For example, while EE may look attractive in terms of its ability to delay emissions, ultimately it will also depend on the costs of achieving EE improvements at those scales; indeed, in our simple simulation model, to achieve more than modest reductions requires rapid reductions in energy demand. An emissions tax would be an efficient policy in the absence of market failures, but a fair evaluation of its costs and benefits relative to the other policies requires taking those market failures and barriers
into account (see, e.g., Fischer and Newell 2008). Nor in our parsing of the policy effects did we allow for emissions prices or energy price changes to induce investments in backstop or energy efficiency improvements or in CCS. In reality, climate policy will be a portfolio of options and responses. The research on intertemporal leakage indicates that this portfolio may need to be somewhat more ambitious than otherwise thought to reach emissions goals, but the efforts are not likely to be undone to the extent indicated by earlier studies.

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School of Business, University of British Columbia (April 8, 2011)
Appendix

Carbon Capture and Sequestration

Carbon capture and sequestration (CCS) has been proposed as one of the few viable options for addressing the green paradox, as it allows for reductions to occur while oil continues to be extracted and consumed (Sinn 2008). As such, it deserves some separate discussion.

We model CCS policy as a mandate that a share $\rho$ of emissions from fuels be captured and stored. The actual emissions from fuel combustion need not be captured and sequestered, a particularly unrealistic idea for transportation fuels; rather, we assume the mandate merely requires an equivalent amount of emissions to be sequestered. Compliance could be achieved either directly (as with the capture of emissions from oil sands upgrading, for example) or indirectly by purchasing offsets or CCS credits (such as from the capture of emissions from coal-fired or gas-fired electricity generation or even aorestation credits).

Let us assume that CCS costs $\kappa$ per unit, so per-unit fuel costs are then $c + (\kappa \rho) \mu + \lambda e^n$. The mandate thus has the same effect on the extraction path and price path as a carbon tax of level $\tau = \kappa \rho$. Hence, a price path in Figure 4 induced by emissions tax $\tau$ would also arise if the policy were instead to mandate that the share $\rho = \tau / \kappa$ of emissions from fossil fuels be sequestered. The two policies would not generate the same cumulative emissions, however. Indeed, the CCS policy would generate $1 - \rho$ times the cumulative emissions.

As a result, the CCS policy is less easy to compare. At a given amount of extraction, the CCS policies would have the same $x_n$ as the cost-equivalent tax. But sequestration means a given amount of extraction is associated with fewer emissions, so meeting the same cumulative target, we have $\rho \kappa < \tau$ (else emissions would be lower) and more extraction than with the tax. If the target is such that extraction is still incomplete, and one is still within region (b), we can prove the ranking using the equilibrium condition (4):

$$B(x_n^{CS}, z_0) = c + \rho \kappa \mu < c + \tau \mu = B(x_n^{tax}, z_0) \text{ so } x_n^{CS} > x_n^{tax}.$$ However, some cumulative targets met by a tax in region (b) might be met with a CCS mandate still in region (a), with full extraction. In this case, we have $B(x_n^{CS}, z_0) = c + \rho \kappa \mu + \lambda e^{-n*}$, which may or may not be less
than $c + \varpi$. In either case, since $c < c + \kappa \rho \mu < c + \kappa \rho \mu + \lambda e^{r*}$, we know that

$$x_{B}^{blend} = x_{B}^{EE} > x_{B}^{CCS},$$

regardless of the region the CCS policy is in for the given target.

CCS cost estimates vary widely, according to the source of the carbon stream being captured (coal-fired power plants being cheaper than industrial sources), the transportation costs, and the sink being used (geological sequestration being cheaper than ocean sequestration or mineral carbonization), as well as monitoring and verification costs (IPCC 2010).\(^{29}\) CCS from oil sands upgrading is likely to be on the costlier end; furthermore, it is limited to the energy used for upgrading, so a mandate of any larger magnitude would require purchasing sequestration credits from other sources. For our purposes, we assume a constant and fixed cost of $100 per ton sequestered, which falls within the admittedly large range of estimates.

While CCS does induce more emissions reductions than the equivalent carbon tax, its susceptibility to intertemporal leakage is not that different in magnitude. It does perform better in the (a) regions, however, particularly between oil shale and oil sands.

\(^{29}\) http://www.ipcc.ch/pdf/special-reports/srccs/srccs_technicalsummary.pdf
Formal Model with n Pools

A1: The Regions if Backstop Cost Reductions are Accelerated

Recall that \( z \geq z_0 \) denotes the investment by the government in accelerating backstop cost reductions. Depending on the setting of \( z \), one of 2n equilibria will arise.

\textbf{m(a):} If pool \( m \leq n \) is the marginal pool, all lower cost pools are exhausted and all higher cost pools remain unexploited. In region \( a \), pool \( m \) itself is exhausted. For any exogenous policy \( z \geq z_0 \), the following 2m equations define the 2m endogenous variables \( (\lambda_1, \ldots, \lambda_m, x_2, \ldots, x_m, x_B) \):

\[
\int_{t=x_k}^{x_{k+1}} D(c_k + \lambda_k e^{rt}) dt = S_k, \quad k = 1, \ldots, m-1;
\]

\[
\int_{t=x_m}^{x_B} D(c_m + \lambda_m e^{rt}) dt = S_m;
\]

\[
c_{k+1} + \lambda_{k+1} e^{rz_{k+1}} = c_k + \lambda_k e^{rz_k}, \quad k = 1, \ldots, m-1;
\]

\[
B(x_B; z) = c_m + \lambda_m e^{rZ}.
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min(c_1 + \lambda_1 e^{rt}, \ldots, c_m + \lambda_m e^{rt}, B(t; z)).
\]
**m(b):** In region \( b \) for pool \( m \), only the fraction \( \theta_m \) of the marginal pool \( m \) is depleted; the remainder is left below ground. For any exogenous \( z \geq z_0 \), the following 2\( m \) equations define the 2\( m \) endogenous variables \( (\lambda_1, \ldots, \lambda_{m-1}, \theta_m, x_2, \ldots, x_m, x_B) \):

\[
\int_{x_k}^{x_{k+1}} D(c_k + \lambda_k e^{\tau t}) dt = S_k, \quad k = 1, \ldots, m - 1; \\
\int_{x_m}^{x_B} D(c_m) dt = \theta_m S_m; \\
c_{k+1} + \lambda_{k+1} e^{\tau x_{k+1}} = c_k + \lambda_k e^{\tau x_k}, \quad k = 1, \ldots, m - 1; \\
B(x_B; z) = c_m.
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min(c_1 + \lambda_1 e^{\tau t}, \ldots, c_{m-1} + \lambda_{m-1} e^{\tau t}, c_m, B(t; z)).
\]

**A2: The Regions if an Emissions Tax is Increased**

Recall that \( \tau \geq 0 \) denotes the emissions tax imposed initially on extractors. Below, we consider the case where the emissions tax rises at the rate of interest.\(^{30}\) Depending on that tax level, one of 2\( n \) equilibria will arise.

**m(a):** If pool \( m \) is the marginal pool, all lower cost pools are exhausted and all higher cost pools remain unexploited. In region \( a \), pool \( m \) itself is exhausted. For any exogenous emissions tax path \( \tau(t) \geq 0 \), the following 2\( m \) equations define the 2\( m \) endogenous variables \( (\tilde{\lambda}_1, \ldots, \tilde{\lambda}_m, x_2, \ldots, x_m, x_B) \):

---

\(^{30}\) To consider the case where the emissions tax is stationary, replace \( e^{\tau t} \) by \( e^{\tau t} \) in each equation.
\[
\begin{align*}
\int_{x_k}^{x_{k+1}} D(c_k + [\tau \mu_k + \lambda_k] e^{rt}) dt &= S_k, \quad k = 1, ..., m - 1; \\
\int_{x_n}^{x_m} D(c_m + [\tau \mu_m + \lambda_m] e^{rt}) dt &= S_m; \\
c_{k+1} + [\tau \mu_{k+1} + \lambda_{k+1}] e^{r x_{k+1}} &= c_k + [\tau \mu_k + \lambda_k] e^{r x_k}, \quad k = 1, ..., m - 1; \\
B(x_B; z) &= c_m + [\tau \mu_m + \lambda_m] e^{r x_B}.
\end{align*}
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min(c_1 + [\tau \mu_1 + \lambda_1] e^{rt}, ..., c_{m-1} + [\tau \mu_{m-1} + \lambda_{m-1}] e^{rt}, c_m + [\tau \mu_m + \lambda_m] e^{rt}, B(t; z)).
\]

**m(b):** In region \( b \) for pool \( m \), only the fraction \( \theta_m \) of the marginal pool \( m \) is depleted; the remainder is left below ground. For any exogenous \( \tau \geq 0 \), the following and \( 2m \) equations define the \( 2m \) endogenous variables \( (\lambda_1, ..., \lambda_{m-1}, \theta_m, x_2, ..., x_m, x_B) \):

\[
\begin{align*}
\int_{x_k}^{x_{k+1}} D(c_k + [\tau \mu_k + \lambda_k] e^{rt}) dt &= S_k, \quad k = 1, ..., m - 1; \\
\int_{x_n}^{x_m} D(c_m + [\tau \mu_m e^{rt}] dt &= \theta_m S_m; \\
c_{k+1} + [\tau \mu_{k+1} + \lambda_{k+1}] e^{r x_{k+1}} &= c_k + [\tau \mu_k + \lambda_k] e^{r x_k}, \quad k = 1, ..., m - 1; \\
B(x_B; z) &= c_m + [\tau \mu_m e^{r x_B}.
\end{align*}
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min(c_1 + [\tau \mu_1 + \lambda_1] e^{rt}, ..., c_{m-1} + [\tau \mu_{m-1} + \lambda_{m-1}] e^{rt}, c_m + [\tau \mu_m e^{rt}, B(t; z)).
\]

**A3: The Regions if Energy Efficiency is Improved**

Recall that \( \varphi > 0 \) denotes energy efficiency measured in energy services per barrel of conventional oil.

**m(a):** If pool \( m \) is the marginal pool, all lower cost pools are exhausted and all higher cost pools remain unexploited. In region \( a \), pool \( m \) itself is exhausted. For any exogenous policy path \( \varphi(t) \geq 0 \), the following \( 2m \) equations define the \( 2m \) endogenous variables

\[
(\lambda_1, ..., \lambda_m, x_2, ..., x_m, x_B)
\]
Given the multipliers in this solution, the equilibrium price path is

\[ p(t) = \min(c_1 + \lambda_1 e^{rt}, \ldots, c_m + \lambda_m e^{rt}, B(t; z)). \]

**m(b):** In region \( b \), only the fraction \( \theta_m \) of the marginal pool \( m \) is depleted; the remainder is left below ground. For any exogenous \( \varphi(t) \geq 0 \), the following \( 2m \) equations define the \( 2m \) endogenous variables \((\lambda_1, \ldots, \lambda_{m-1}, \theta_m, x_2, \ldots, x_m, x_B)\):

\[
\int_{t=x_B}^{x_B} \frac{1}{\varphi(t)} D \left( \frac{c_k + \lambda_k e^{rt}}{\varphi(t)} \right) dt = S_k, \quad k = 1, \ldots, m-1;
\]
\[
\int_{t=x_m}^{x_m} \frac{1}{\varphi(t)} D \left( \frac{c_m + \lambda_m e^{rt}}{\varphi(t)} \right) dt = S_m;
\]
\[
c_k + \lambda_k e^{rt} = c_k + \lambda_k e^{rt}, \quad k = 1, \ldots, m-1;
\]
\[
B(x_B; z_0) = c_m.
\]

Given the multipliers in this solution, the equilibrium price path is

\[ p(t) = \min(c_1 + \lambda_1 e^{rt}, \ldots, c_m + \lambda_m e^{rt}, c_m, B(t; z)). \]

**A4: The Regions under a Blend Mandate**

Recall that \( \beta \in [0,1) \) denotes the minimum share of energy needs that must be met by the the backstop technology (an obvious example is the ethanol blending requirement). This requirement has two effects: 1) it imposes an additional cost on fossil fuels, in the form of an implicit tax equal to the cost of backstop fuel required per unit of fossil fuel, and 2) it replaces a share of fossil fuel with the backstop in overall energy consumption.
m(a): If pool \( m \) is the marginal pool, all lower cost pools are exhausted and all higher cost pools remain unexploited. In region \( a \), pool \( m \) itself is exhausted. For any exogenous policy path the following \( 2m \) equations define the \( 2m \) endogenous variables \((\lambda_1, \ldots, \lambda_{m-1}, x_2, \ldots, x_m, x_B)\):

\[
\int_{t=x_k}^{x_k} (1-\beta)D((1-\beta(t))(c_k + \lambda_k e^{\gamma_k}) + \beta(t)B(t;z_0)) dt = S_k, \quad k = 1, \ldots, m-1;
\]

\[
\int_{t=x_{m-1}}^{x_m} (1-\beta)D((1-\beta(t))(c_m + \lambda_{m-1} e^{\gamma_{m-1}}) + \beta(t)B(t;z_0)) dt = S_m;
\]

\[
c_{k+1} + \lambda_{k+1} e^{\gamma_{k+1}} = c_k + \lambda_k e^{\gamma_k}, \quad k = 1, \ldots, m-1;
\]

\[
B(x_B; z_0) = (c_m + \lambda_m e^{\gamma_m}).
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min[(1-\beta(t))(c_1 + \lambda_1 e^{\gamma_1}) + \beta(t)B(t;z_0), \ldots, (1-\beta(t))(c_m + \lambda_m e^{\gamma_m}) + \beta(t)B(t;z_0), B(t;z_0)].
\]

m(b): In region \( b \), only the fraction \( \theta_m \) of the marginal pool \( m \) is depleted; the remainder is left below ground. For any exogenous path \( \beta(t) \in [0,1] \), the following \( 2m \) equations define the \( 2m \) endogenous variables \((\lambda_1, \ldots, \lambda_{m-1}, \theta_m, x_2, \ldots, x_m, x_B)\):

\[
\int_{t=x_k}^{x_k} (1-\beta(t))D((1-\beta(t))(c_k + \lambda_k e^{\gamma_k}) + \beta(t)B(t;z_0)) dt = S_k, \quad k = 1, \ldots, m-1;
\]

\[
\int_{t=x_{m-1}}^{x_m} (1-\beta(t))D((1-\beta(t))(c_m) + \beta(t)B(t;z_0)) dt = \theta_m S_m;
\]

\[
c_{k+1} + \lambda_{k+1} e^{\gamma_{k+1}} = c_k + \lambda_k e^{\gamma_k}, \quad k = 1, \ldots, m-1;
\]

\[
B(x_B; z_0) = c_m.
\]

Given the multipliers in this solution, the equilibrium price path is

\[
p(t) = \min[(1-\beta(t))(c_1 + \lambda_1 e^{\gamma_1}) + \beta(t)B(t;z_0), \ldots, (1-\beta(t))(c_m + \lambda_m e^{\gamma_m}) + \beta(t)B(t;z_0), B(t;z_0)].
\]

A5: The Regions under a CCS Mandate

Recall that \( \rho(t) \) is the share of fossil fuel emissions that must be captured and sequestered, either directly (as with oil sands upgrading) or indirectly (as with buying credits from coal-fired generation with CCS or aforestation). These activities are assumed to cost \( \kappa \) per unit.
**m(a):** In region $a$, the following $2m$ equations define the $2m$ endogenous variables 

$$(\lambda_1, \ldots, \lambda_m, x_2, \ldots, x_m, x_B) :$$

\[
\int_{r=x_k}^{x_{k+1}} D(c_k + \rho(t) \kappa \mu_k + \lambda_k e^{\rho(t)e^t}) dt = S_k, \quad k = 1, \ldots, m-1;
\]

\[
\int_{r=x_m}^{x_B} D(c_m + \rho(t) \kappa \mu_m + \lambda_m e^{\rho(t)e^t}) dt = S_m;
\]

\[
c_{k+1} + \rho(x_{k+1}) \kappa \mu_{k+1} + \lambda_{k+1} e^{\rho(x_{k+1})e^t} = c_k + \rho(x_k) \kappa \mu_k + \lambda_k e^{\rho(x_k)e^t}, \quad k = 1, \ldots, m-1;
\]

\[
B(x_B; z) = c_m + \rho(x_B) \kappa \mu_m + \lambda_m e^{\rho(x_B)e^t}.
\]

Given the multipliers in this solution, the equilibrium price path is

\[\rho(t) = \min(c_1 + \rho(t) \kappa \mu_1 + \lambda_1 e^{\rho(t)e^t}, \ldots, c_m + \rho(t) \kappa \mu_m + \lambda_m e^{\rho(t)e^t}, B(t; z)).\]

Total emissions are

\[E = \sum_{k=1}^{m} \mu_k \int_{r=x_k}^{x_{k+1}} (1 - \rho(t)) D(\rho(t)) dt.\]

The remaining \(\sum_{k=1}^{m} \mu_k S_k - E\) is sequestered.

**m(b):** In region $b$, the following $2m$ equations define the $2m$ endogenous variables 

$$(\lambda_{-1}, \ldots, \lambda_{m-1}, \theta_m, x_2, \ldots, x_m, x_B) :$$

\[
\int_{r=x_k}^{x_m} D(c_k + \rho(t) \kappa \mu_k + \lambda_k e^{\rho(t)e^t}) dt = S_k, \quad k = 1, \ldots, m-1;
\]

\[
\int_{r=x_m}^{x_B} D(c_m + \rho(t) \kappa \mu_m) dt = \theta_m S_m;
\]

\[
c_{k+1} + \rho(x_{k+1}) \kappa \mu_{k+1} + \lambda_{k+1} e^{\rho(x_{k+1})e^t} = c_k + \rho(x_k) \kappa \mu_k + \lambda_k e^{\rho(x_k)e^t}, \quad k = 1, \ldots, m-1;
\]

\[
B(x_B; z) = c_m + \rho(x_B) \kappa \mu_m.
\]

Given the multipliers in this solution, the equilibrium price path is

\[\rho(t) = \min(c_1 + \rho(t) \kappa \mu_1 + \lambda_1 e^{\rho(t)e^t}, \ldots, c_{m-1} + \rho(t) \kappa \mu_{m-1} + \lambda_{m-1} e^{\rho(t)e^t}, c_m + \rho(t) \kappa \mu_m, B(t; z)).\]