Credit Derivatives
An Overview and the Basics of Pricing

Master Thesis

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Abstract

Credit exposure is a source of risk that is present in almost all financial transactions and credit derivatives provide the most innovative method to manage this risk. Among the different categories of credit derivatives, this thesis concerns only default products such as credit default swap, collateralised debt obligation and credit linked note. The thesis aim to describe the most commonly used credit derivatives and how a Swedish bank may use these. Also, the basics of pricing with the so called reduced form credit pricing model will be explained and how this model can be calibrated from real market data. Although the origin of credit derivatives was the need of a new risk-management tool the main use of credit derivatives in Sweden is rather investment trading with the purpose to earn speculative profits.

Key words: Credit Derivatives, Credit Default Swaps, Collateralised Debt Obligation, Credit Linked Note, Credit Risk Pricing Models, Reduced Form Model, Default Probability, Intensity of Default
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1. Background

Credit risk can be defined as the changes in value due to unexpected changes in the credit quality of issuers or counterparties. A credit derivative is an instrument which pay-off is related to the credit worthiness of a borrower. It provides protection against credit risk in return for a fee paid to the party who assumes the risk. The global market for credit derivatives barely existed ten years ago and today the British Bankers’ Association predicts that it will rise beyond eight trillion USD by the end of 2006. Banks are the largest net sellers of credit risk while insurance companies and hedge funds, along with banks, are net buyers.

Graph 1:1  
Credit Derivatives Global Notional Outstanding
Source: Celent home page

Credit exposure is a source of risk that is present in almost all financial transactions and credit derivatives provide an innovative method to manage this risk. The instruments transfer credit risk between parties, which allow banks to diversify their loan portfolios and their counterparties to earn higher returns. A result of the transfer of credit risk is that the exposure spreads over the economy, rather than remain in banks. In order to diversify

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1 http://www.bba.org.uk/bba/jsp/polopoly
2 http://www.celent.com/PressReleases/20040130(2)/CreditDerivatives.htm
banks can also buy protection from other banks which means that the credit risk is still in the banking system, but the exposure to a single borrower is reduced.

An example of the effect of credit derivatives is the default of Enron, WorldCom and the Argentina government in 2002. As the quality of their assets deteriorated, the lenders bought credit default swaps as protection, because of that the costs fell on a broader group of banks and financial institutions than would otherwise be the case and hence the defaults did not have the devastating effect on big banks’ balance sheet that might have been expected. (The Economist 2002)
2. Problem Discussion

In the relatively new credit derivatives market there are numerous instruments with different characteristics. Different needs and motives for the use of these instruments are factors that decide which instrument is of interest to a certain investor. In order to define the specific features of the instruments a description is needed.

Which are the most commonly used credit derivative instruments and what are their characteristics?

The original driving force for the development of the credit derivatives market came from banks in USA with very large exposure to a single company. There was a need for the possibility to control credit exposure without refusing loans to a good customer. Today when the credit derivatives market has grown rapidly for a number of years, there are different practical applications for the contracts such as credit risk management and trade with the purpose to make speculative profits.

How can a Swedish bank use and price credit derivatives?

The importance of correct assessment of credit risk can not be emphasized enough. Risk management is also the foundation of pricing models for credit derivatives. To model credit risk and decide the appropriate risk premium associated with a certain derivative contract the so called reduced form or intensity based model can be used. In this framework, the intensity can be constant, deterministic time dependent or a stochastic process.

What is the basic idea of a reduced form pricing model and how sensitive is the reduced form model for changes in the default intensity $\lambda_t$?
3. Outline

This thesis begins with a background review of the subject, the original purpose and history of credit derivatives. The first chapter gives various examples of real events where credit derivatives have served their purpose and eased the financial loss at some critical credit events. In the previous chapter, entitled Problem Discussion, the main questions of this thesis are highlighted. These questions are preceded by a brief discussion about the difficulties that surrounds credit derivatives.

In order to answer the questions raised in this thesis in a manner as straightforward as possible, each subject is considered separately in the subsequent chapters. The theoretical framework presents the general idea, characteristics and applications, of the most commonly used credit derivatives along with a review of the notion of credit risk and credit spread. To be able to answer the question on how Swedish banks use and price credit derivatives an interview was conducted with Handelsbanken. The result of this meeting is presented in the chapter entitled Qualitative Empirical Result.

The models used to price credit derivatives is reviewed and explained in the fifth chapter. A simple version of a reduced form model is used to explain the basic drivers of the model and to illustrate the sensitivity of these elements. This is followed by a conclusion where the main questions raised in this thesis is answered.
4. Theoretical Framework

4.1 Credit Derivatives

Credit derivatives can be divided into two different classes, namely default and spread products. The first is an instrument which payoff depends on a credit event such as payment default and bankruptcy, a downgrade in credit ratings below a given threshold or changes in the credit spread changes the payoff of the later product. (Herbertsson 2005) This thesis will focus on default products.

The credit event is defined with respect to a reference credit which can be a loan, a bond or a basket of assets to name a few. If a credit event occurs a default payment has to be made, this is the defining feature of most credit derivatives. The seller or buyer in a credit derivatives transaction usually refers to the seller or buyer of protection and that is also the definition that will be used in this paper. (Schönbucher 2000)

In the following sections the features of some of the most commonly used credit derivatives will be explained. Basket credit default swaps will be described because of their significance in the credit derivatives market, otherwise there will be focus only on the single name credit derivatives.

4.1.1 Credit Default Swaps (CDS)

The traditional single name credit default swap is a payment by one party to another party, in exchange for a default payment, if a credit default event on the reference asset occurs within a specific time period, typically five years. In return for the protection, the buyer pays the seller a premium, which is usually paid to the seller over time in three months intervals. The default payment will often be the difference between the original price of the reference asset and the recovery value of the reference asset, i.e. the credit loss suffered at the default event. If the credit event does not occur before the maturity of
the credit default swap the default protection seller has no obligation. Default protection can be purchased on a loan, a bond, sovereign risk due to cross border commercial transactions, or even credit exposure in a cross currency swap transaction. Credit protection can be linked to an individual credit or to a basket of credits. (Tavakoli 1998)

4.1.2 Basket Credit Default Swaps

Just as a portfolio manager can purchase protection on a single credit, protection can be purchased on a basket of credits for $T$ years. If the basket credit default swap has a first to default structure, the swap will be terminated at $T$ or when the first of the basket’s credits defaults and the protection seller has to make a payment to the protection buyer. Other common structures for basket credit default swaps are second to default and third to default. (Tavakoli 1998)

4.1.3 Collateralised Debt Obligation

A collateralised debt obligation is an asset-backed security where the underlying asset is a portfolio of bonds or bank loans, exposed to credit risk. The rationale for a financial institution to create collateralised debt obligations is to manage its’ credit exposure and, therefore, reduce the amount of capital required according to current regulations, as well as to earn profits. A securitisation of the debt instruments may also involve an improved liquidity of the sometimes illiquid loans and bonds. (Duffie & Singleton 2003)

The portfolio risk and hence the credit risk, is transferred to a special purpose vehicle which usually divides the interest income and principal repayments into three or four different securities, commonly referred to as tranches. These tranches are divided depending on the level of default protection, to reflect the priority at which the buyer of a tranche receives payment at the occurrence of a credit event.

As the underlying assets default the payments to the owners of the equity, junior, mezzanine and senior tranches, in given order, are cancelled. The probability of default is lower for the senior and mezzanine tranches, compared to the original portfolio, and
higher for the equity and junior tranches. With a collateralised debt obligation, medium-quality debt can be transformed into a high quality instrument. The protection buyer’s risk exposure depends on the default correlation between the different underlying assets; the lower the correlation the higher the tranches will be rated. (Hull 2003)

Figure 4:1  
*Simple CDO Structure*

Source: Dagens Industri, 2005-05-02

There are two types of collateral debt obligations depending on whether or not the portfolio is transferred to the special purpose vehicle. The original collateral debt obligation structure involves a transfer of the underlying assets from the protection buyer to the protection seller.

The unfunded, or synthetic, collateral debt obligation consist of credit default swaps rather than bonds or loans. In this case, the special purpose vehicle issues credit default swaps to the end seller of protection and invest the proceeds in high-quality collateral securities, such as government or mortgage bonds. The interest received from these high quality investments is transferred to the end-seller along with the premium on the default swaps. (Rule 2001)

4.1.4 Credit Linked Notes

Credit derivatives can be embedded in notes, which are generally referred to as credit-linked notes. A credit linked note can be a combination of a credit derivative and a bond issued by the counterparty. It allows the issuer to transfer a specific credit risk to credit investors. Credit linked notes can be created through a Special Purpose Company, or
trust, which is collateralised with AAA-rated securities. Investors buy securities from a trust that pays a fixed or floating coupon during the life of the note. At maturity, the investors receive par unless the referenced credit defaults, in which case the investor receive an amount equal to the recovery rate.\(^3\)

The trust enters into a default swap with a deal arranger and in case of default, the trust pays the dealer par minus the recovery rate in exchange for an annual fee which is passed on to the investor in the form of a higher yield on the notes. Under this structure, the coupon or price of the note is linked to the performance of a reference asset. It offers borrowers a hedge against credit risk and gives investors a higher yield on the note for accepting exposure to a specified credit.\(^4\)

Unlike many credit derivatives, the credit linked note is not an off-balance sheet transaction and this allows investors, that have regulatory or contractual restrictions on their use of derivatives to invest in credit linked notes.\(^5\)

### 4.1.5 Total rate of return swaps

A total rate of return swap allows an investor to enjoy all of the cash flow benefits of a security without actually owning the security. The swap is a financial contract between a total-return payer and a total-return receiver where the payer pays a total return of a reference security and receives a form of payment from the total-return receiver. The payment is often floating rate, a spread to LIBOR, and the reference security can be indices, bonds, loans, real estate receivables, lease receivables or commodities. (Tavakoli 1998)

The total rate of return payer is the legal owner of the reference asset and the total rate of return swap is an off-balance sheet transaction, which implies that the reference asset does not appear in the balance sheet of the receiver. During the time of the transaction the

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\(^3\) [http://www.handelsbanken.se/shb/IneT/StartSv.nsf/FrameSet?OpenView&Id=Z_HMC](http://www.handelsbanken.se/shb/IneT/StartSv.nsf/FrameSet?OpenView&Id=Z_HMC)

\(^4\) [http://www.investopedia.com/terms/c/creditlinkednote](http://www.investopedia.com/terms/c/creditlinkednote)

\(^5\) ibid
total rate of return receiver has a synthetic long position in the market risk as well as the credit risk of the reference asset. In the case of default, the investor may make a net payment of the difference between the price of the reference security at the beginning of the transaction and at the time of default. (Tavakoli 1998)

The total rate of return swap may have very important credit implications for both the payer and the receiver of the swap, but it is primarily used as a financing vehicle. The key reason for the investor to enter into this transaction is to take advantage of leverage since there is no initial cash payment. (Tavakoli 1998)

4.2 Applications of Credit Derivatives

One purpose of credit derivatives is to transfer credit risk from for example a bank, which wants to reduce its exposure to a certain credit, to an investor who wants to earn income. There is also the purely speculative reason to get involved in trades with these kinds of instruments, where the idea is to earn excess returns. Banks are the largest group of credit derivatives users who use the derivatives to free up or manage credit lines, manage loan exposure without needing the consent of debtor, manage regulatory capital or exploit comparative advantages in cost of funding. The management of credit exposure involves reduction of credit concentration through basket structures, easier diversifications of credit risk and the direct hedge of default risk. The party that assumes the risk can either trade a view of the credit quality of the reference asset or make arbitrage profits of mispriced defaultable bonds. (Schönbucher 2000)

4.3 Credit Risk

Credit risk is one source of market risk that is present in almost all financial transactions. In contrast to other sources of market risk, the market for credit has to a higher extent, intrinsic adverse selection and moral hazard. Due to the lack of transparency and the fact
that larger loans are likely to present incentives for borrowers to engage in more risky activities. These two market imperfections stress the importance of solid models to measure and diversify credit risk. (Duffie & Singleton 2003)

Compared to the information underlying market risk, such as price volatility, credit risk is dependant on entirely different sources. Central to the distribution of credit losses is the probability of default. This likelihood of a credit downgrade, payments overdue or any other failure to honour the financial agreement is of utmost importance to estimate in order to measure the credit risk. (Giesecke 2004)

There are several different ways to estimate the probability of default. Real-world probabilities are backed out from historical data, which in general are lower than the risk-neutral probabilities derived from bond prices. Another way to obtain the probability that a company will default at some time in the future involves the use of information from equity prices. (Hull 2003)

To reduce the losses due to counterparty defaults, certain risk reducing measures can be taken, such as netting, collateralisation or contractual downgrade triggers. At the event of a default on one contract the netting clause, a common element in credit contracts, allow the loan provider to cancel all outstanding contracts with that specific counterparty. Further, the expected credit loss can be reduced by the provider of the credit, if collateral is paid by the borrower at the issuance of a contract, or if a downgrade trigger is included in the contract which will cancel the credit if the borrower receives an insufficient credit rating. (Hull 2003)
4.4 Credit Spread

Credit spreads can be used as a measure of the risk premium connected with credit risk and can consequently be described as a component of the spread between treasury securities and non-treasury securities that are identical in all respects except for quality rating.\(^6\)

To assess the price of risk in the credit derivatives market, credit default swap spreads have been compared to bond spreads and empirical findings confirm that the credit default swap spreads and bond spreads move together in the long run. The theoretical foundation is based on the no-arbitrage conditions which suggest that a credit default swap can be replicated synthetically by shorting a par fixed coupon on the same reference entity with the same maturity and investing the proceeds in a par fixed coupon risk-free note. The credit default swap spread should be approximately equal to the credit spread, i.e. yields minus the risk-free rate, of the underlying bonds. (Zhu 2004)

If the constant premium that the protection buyer pays is greater than the bond’s coupon payments less the risk-free rate, an investor can sell the credit default swap in the derivatives market, buy a risk-free bond and short the corporate bond in the cash market, and make arbitrage profits. If the premium, i.e. the credit default swap spread is less, the reverse strategy would yield arbitrage profits. (Zhu 2004)

A useful indicator of price discrepancies is the basis spread, which is defined as the five-year credit default swap spreads minus the five-year bond spreads (bond yields minus the risk-free rate). Although the two credit risk prices tend to be equal over time, there are price differences in the short run. (Zhu 2004)

Credit spreads that are implied by the reduced form pricing model are given by the intensity of default. The premium, which is a function of the intensity, corresponds to the

\(^6\) http://www.investopedia.com/terms/c/creditspread.asp
expected loss of one dollar over an infinitesimal horizon. In contrast to the reduced form model, the credit spreads implied by the structural models, goes to zero when the time to maturity goes to zero. The intuition behind this result comes from the fact that the structural models assume that default is predictable within a very short time period prior to the default and hence there is no premium required for uncertainty. (Giesecke 2004)
5. Market Study

5.1 Interview

This section concerns the information received from a dialogue with Niklas Felixson who is the chief dealer for credit derivatives at Handelsbanken Capital Markets. The questionnaire which served as a base for an interview with Mr Felixson can be found in the appendix.

5.1.1 Instruments

The most traded credit derivatives are single name credit default swaps, where the outstanding volume for the bank amounts to approximately 0.5 billion euros of which Handelsbanken is a net buyer of protection. Majority of the credit default contracts comply with what the International Swaps and Derivatives Association (ISDA) defines as standard with regard to definitions of credit event, estimated recovery rate and maturities. The typical nominal amount on a credit default swap is between 5 and 10 million Euros. The currency, in which the credit default swaps are traded by the bank, is most commonly Euros followed by US dollars.

In addition to the credit default swaps, Handelsbanken trade a lot with index products, these products has a typical nominal amount between 10 and 100 million euros. Credit linked notes are the basket product with largest trading volume and this is also a product with strong growth, not least internationally.

It is not common practise to modify the contracts for customers, but the possibility exists. Apart from the standard contracts, the bank develops products that can meet the investors’ needs and this force drives the credit derivatives market.
5.1.2 Applications

Handelsbanken has a tradition in dealing with corporate bonds and credit default swaps can be seen as a complement to these. The credit default swaps provide an easy way for the bank to take a short position and short speculative positions are the main application for credit derivatives at Handelsbanken. In Sweden, credit derivatives are used as trading instruments rather than to manage the bank’s overall credit risk exposure. However, the credit default swaps provide an easy way to measure credit risk, since the bond market is transparent and the loan market is not.

The single name products are foremost traded between banks, because of the correlation problems that can arise when certain industry companies for example sells protection on another company in the same industry.

5.1.3 The Market

The global market for credit derivatives is today estimated to 7 trillion US dollars worth of outstanding contracts, but the size of the Swedish market is difficult to estimate because there is no available data. Approximately 70 percent of the market is what Niklas Felixson calls known names, companies that are rated by Moody’s and S&P and these names are also part of the index products that are traded. Although the development of the Swedish credit derivatives market is late compared to USA and some European countries, continued rapid growth can be expected. Some reasons for the late development in Sweden are that the banks did not have systems to handle the product or employees who are familiar with credit derivatives.

5.1.4 Secondary Market

There is a liquid secondary market for credit derivatives, especially for standard credit default swaps, consequently the spread between buy and sell prices are narrow. The best liquidity can be found for standard credit default swaps with a five year maturity, for non standardised contracts there exist a market, but liquidity is low.
Bloomberg is the main information system used by credit derivative dealers at Handelsbanken and they provide methods for deciding objective prices for both parties in the secondary market.

5.1.5 Counterparty

Most of Handelsbanken’s trades are with large European banks as counterparty, the reason for this is that they are large actors in the credit derivative market, but even more importantly the correlation problems are less than if Handelsbanken would have Swedish banks as counterparty. However, correlation can still be a problem when the reference entity is a Swedish industrial company, since there is a history of correlation between Handelsbanken and the Swedish industry. When there are credit events in the industrial companies, the bank’s finances are usually poor as well. It is important for the buyer of protection that there is as little correlation as possible between the counterparty and the underlying instrument.

5.1.6 Investors

Niklas Felixson names three main categories of investors namely private savers, small pension funds and institutional investors. Basket products, which include credit default swaps as building blocks, are the appropriate alternative for private savers who are advised to hold these assets until maturity, typically three to five years. Private savers interest in assuming credit risk is a recent event, but the trend is increasing interest for structured products. Most people today are used to assume the market risk associated with equity investment and the step to structured products is natural.

Smaller pension funds and other mid size investors are undoubtedly interested in the investment opportunities that exists in the credit derivatives market. Historically they invest in specific corporate bonds and now the possibility of obtaining customised contracts exists. Institutional investors on the other hand have different requirements, since they hold positions only a short time. Liquidity must be high in a whole class of assets and trading costs must be low, in order to interest the institutional investors. Today
index products are the most liquid credit derivatives products and hence the best alternative for institutional investors.

5.1.7 Pricing Models
Handelsbanken do not use models on a daily basis to price credit derivatives. Since there is a liquid secondary market for the most commonly traded contracts, the market prices are assumed to be correct. When the actual market price differs from the theoretical price, which can be generated by the use of a pricing model, there is always a good reason for this according to Mr Felixson. In cases where the secondary market does not provide liquid prices for the credit derivative, the corporate bond spread can be used as reference.

If a new name, a credit derivative on a reference entity that has not been traded previously, is to be priced, credit rating is considered first. In addition the bank may have internal rating on the company in question and the bank’s credit analysts will be consulted. Rather than to use fundamental analysis to price specific credit derivatives, the focus is on pricing correlation between the instruments. To correctly price dependence is of increasing significance, especially since there is extensive increase in the market for basket products.

5.1.8 Recovery Rates
There is a difference between the estimated recovery rate that is used to price a credit derivative and the actual recovery rate when default has occurred. Handelsbanken who mostly deals with standardised products, use market practise which almost always is 40 percent of the nominal amount. If another recovery rate is used to price an instrument, such as a customised basket product, the client is explicitly told. Moody’s and S&P provide information about historical recovery rates for different products, but if a new product is to be priced assumptions have to be made.

The true recovery rate is a concern for the seller of protection who is obliged to pay the nominal amount in return for the asset, upon the occurrence of a credit event, no matter
what price the asset currently is traded at. This is true when a physical delivery of the asset is contracted, but when there is cash settlement the conduct is different. A panel that consists of a number of actors in the market, which is stipulated in the contract, are consulted and they decide what market value should be.
6. Credit Risk Pricing Models

When modelling credit risk two main classes of models exist, the structured and the reduced form. The most significant difference between these models is the information set that the modeller is assumed to have access to. To use the structural model for credit risk valuation detailed information about the company is required, information typically held by managers. On the other hand, reduced form models assume that the modeller have access to the relevant information set available in the market which implies that the firm’s time of default is unpredictable. (Jarrow & Protter 2004)

Instead of view the structured and reduced form pricing models as competing and disconnected, it is possible to show that the reduced form model can be obtained from the structural model if a smaller, incomplete, information set is used. Since the asset value process is unobservable by the market it is easier to use the reduced form pricing model. (Jarrow & Protter 2004)

Further, the reduced form model does not formulate any economic arguments on why a firm defaults, the default event and its stochastic structure is exogenously given. Default is unpredictable and in order to capture this feature, the intensity based approach models default as some unpredictable Poisson-like event. The Poisson process is a certain type of stochastic process, where the described default event, can not be predicted even a moment before a default occurs. (Giesecke 2002)

6.1 Structural Credit Pricing Models

The structural, or firm based, pricing models are based upon the assumption that corporate liabilities are contingent claims on assets of a firm. Default will occur when the firm is not able to meet its obligations which imply that the firm’s market value is the essential element of the uncertainty driving credit risk.
The idea behind these models is that the value of the firm is assumed to follow a stochastic process. This process is the driving force behind the values of all securities issued by the firm, all claims (debt and equity) on the firm can be seen as options with the firm’s value as underlying asset. Default can actually be triggered in two ways, either the debt is paid at the maturity of the contract, and default can therefore only occur at maturity, or default is triggered as soon as the firm’s value falls below a certain barrier which often represents the liabilities. (Schönbucher 2000)

Graph 6:1  
*Structural Pricing Model*

Source: Duffie & Singleton (2003, p 54)

The structural approach goes back to the works of Black & Scholes (1973) and Merton (1974). In these approaches the debt of the firm can be seen as a European option and default can only occur at maturity of the debt. The barrier approach was introduced in 1976 by Black and Cox, where valuation of various corporate bonds and bond covenants was also illustrated. This was further developed by Nielsen et al. which introduced a stochastic default barrier in 1993. In 1977, Geske modelled defaultable coupon debt as a compound option on the firm’s value, where defaults are triggered when the value is insufficient to meet the coupon obligations. (Duffie & Singleton 2003)

The problem of low credit spreads for short time intervals, close to maturity, was dealt with by Schönbucher (1996) and Zhou (1997). Since the probability for a continuous
diffusion process to reach the default barrier in a very short time interval is extremely small, this will imply low credit spreads. The resulting equation is solved by a numerical finite-difference algorithm, proposed by Schönbucher, or a Monte-Carlo simulation according to Zhou (1997).

Commercially, the KMV model introduced by Kealhofer, McQuown and Vasicek is probably the most credited. The KMV model use default likelihoods from historical data along with the structural approach’s key indicator, distance to default, as definition for credit quality (Schönbucher 2000).

To date, there are few empirical studies of the performance of the structural models for pricing defaultable bonds. According to research from Crosbie, the KMV model gives a better result compared to the usage of the firm’s rating for credit default expectations. This is also supported in empirical studies by Kwan (1996), Jones et al. (1984) and Sarig and Warga (1989). These results imply that the connection between the share prices and defaultable bonds is far too weak to be able to implement a successful hedging strategy based on it (Schönbucher 2000).

6.2 Reduced Form Credit Pricing Models

Because of the limitations associated with the structural credit pricing model there was a need for a new model. In 1995 the first reduced form credit model was developed by Jarrow and Turnbull and since then several developments of this model has been made. The reduced-form pricing model, which is also called the intensity based model, assumes that default occurs without warning at an exogenous default rate, or intensity. Instead of ask why the firm defaults, the intensity model is calibrated from market prices. (Schönbucher 2000) The calibration of default intensity can be compared with how implied volatilities are backed out from market prices when the Black Scholes pricing formula is used to price equity options.
6.2.1 Survival Probability and Default Intensity

The general assumption is that the present value of the cost of default is equal to the difference between the return on a risk-free and a corporate bond. This follows the no-arbitrage condition that there cannot be excess return without a corresponding amount of risk. (Hull 2003)

The reduced form pricing models, which are the focus of this paper, assume that the default arrival follows a Poisson process. In the classic Poisson-arrival model, the arrival risk is independent over time and, although default may be only an instance from the present time.

The plain version of the reduced form pricing models defines default as the first arrival time, \( \tau \), of a Poisson process with constant arrival rate \( \lambda \), called intensity. This implies that the probability of survival, up until time \( t \), is \( p[\tau > t] = e^{-\lambda t} \) and that the probability of default, for a small time period \( \Delta t \), is approximately equal to \( \Delta t \lambda \) conditional of survival up to time \( t \), i.e. (Duffie & Singleton 2003)

\[
p[\tau \in (t, t + \Delta t) | \tau > t] \approx \lambda \Delta t.
\] (1)

This relationship also holds for deterministic time dependent intensities, i.e. \( \lambda = \lambda(t) \), which implies that \( p[\tau > t] = e^{-\int_0^t \lambda(s) ds} \). In this case, \( \tau \) is modelled as the time of the first jump of an inhomogeneous Poisson process with intensity \( \lambda(t) \). (Schönbucher 2003)

In general, \( \lambda(t) \) is modelled as a stochastic process to reflect the fact that the default intensity of an obligor may fluctuate randomly over time.
Then one can show that

\[ p[\tau > t] = E \left[ e^{-\int_{0}^{t} \lambda(s) ds} \right]. \quad (2) \]

Note that in order to compute \( p[\tau > t] \) in the equation (2), the distribution of \( \lambda(s) \) for all \( s \leq t \), must be known. (Schönbucher 2003)

### 6.2.2 Recovery Rate

The net compensation that the protection buyer receive from the protection seller at the event of a default is denoted the recovery rate. In the simplest form of reduced models the recovery rate is said to be a fraction of the market value of the bond just before default or a specified expected fraction of face value. The latter definition of the amount received from a defaulted bond implies that, at the event of a default, the firm is liquidated and the bondholders obtain an appropriate amount determined by the priority of their holdings. This is not a fully realistic approach as bankruptcy courts do not always honour this priority. (Duffie & Singleton 2003)

The fractional recovery of market value-approach sets the recovery rate to a specified fraction of the market value of the defaulted bond determined by the time at which the default occurs. (Duffie & Singleton 2003)

### 6.2.3 Simple Version of a Reduced Form Credit Pricing Model

In this section a simple example of the reduced form model, that illustrates the default swap premium as a function of the intensity, is presented. This equation consists of a default leg and a fee (premium) leg and by equating the default leg with the premium leg the premium for the credit default swap at a specific time will be obtained. The
arguments presented here is standard in the credit risk literature, see for example Bluhm et al. (2003).

When valuing the fee leg, \( f \), the credit default swap premium, \( Q \), is supposed to be a fraction of the hypothetical basis points per year that is paid at dates \( s_1 < s_2 < \ldots < s_n = T \) with \((s_i - s_{i-1})\) representing the interval between the payments, in other words the protection buyer pays \( Q(s_i - s_{i-1}) \) basis points (bp) of the nominal amount at each payment period, typically three months. The probability that a default has not yet occurred at time \( s_i \) will be denoted \((1 - p(s_i))\) and this implies that the present value of the spread payments (fee leg) is given by

\[
f = \sum_{i=1}^{n} Q(s_i - s_{i-1})(1 - p(s_i))B_{0}^{s_i}.
\]  

Bluhm et al. (2003) assume that the recovery rate \( R \) is a random variable that is independent of the default. In order to simplify the model the recovery rate is assumed to be constant. The present value of the loss upon default is described by the equation

\[
d = (1 - R) \int_{0}^{T} B_{0}^{t} F(dt).
\]  

In equation (4), the term \( F(dt) \) describes the probability that a default will happen within an infinitesimal time interval \( dt \). The default probability will then be multiplied by the discount factor \( B_{0}^{t} \). The model can be adjusted to discrete time by letting \( dt \) be substituted by \( \Delta t \) and hence the probability of default at time \( t \) will be replaced by the probability of default during the time period \( t_{i} - t_{i-1} \). These adjustments will replace the expression \( F(dt) \) with \([p(t_{i}) - p(t_{i-1})]\), which is the difference between the probability of default at time \( t_{i} \) and time \( t_{i-1} \). Hence, the integral in equation (4) can be approximated by a summation as,
illustrated by equation (5). The values from the integral and the values generated by the summation will converge as the time interval goes to zero.

\[
(1 - R) \sum_{i=1}^{m} \left[ p(t_i) - p(t_{i-1}) \right] B_{0}^{t_i} 
\]  

(5)

The fair default swap premium, Q, is determined so that the present values in equation (3) and equation (5) are equal. Hence, Q is then given by the equation

\[
Q = \frac{(1 - R) \sum_{i=1}^{m} \left[ p(t_i) - p(t_{i-1}) \right] B_{0}^{t_i}}{\sum_{i=1}^{n} (s_i - s_{i-1})(1 - p(s_i)) B_{0}^{s_i}}. 
\]  

(6)

6.3 The Sensitivity of a Reduced Form Credit Pricing Model

The aim is to plot a set of curves for the credit default swap premium for different parameters in the default intensities from equation (6). To be able to plot the desired curve some modifications is made to suit our needs and prerequisites. These modifications are discussed and explained below.

When it comes to the expected value of the recovery rate, we will use a fixed value of 40 percent as this is the most common figure used when trading with single name credit derivatives, according to Niklas Felixson\(^7\) and Handelsbanken. For the discount factor \(B_{0}^{t_i} = e^{-r_t}\) we will use a fixed risk-free interest rate of 2.7 percent which is the notation for the five year government bond\(^8\).

\(^7\) Niklas Felixson Chief Dealer Credit Derivatives Handelsbanken, meeting 2005-05-02
\(^8\) www.rgk.se 2005-05-06
As the credit default swap payments seldom are as frequent as the discrete dates set for default we will denote these two differently. The time intervals that concern the credit default swap payments will be denoted $s$ and those that concern default $t$, they must nevertheless coincide.

The intensity will be determined by the two constants $a$ and $c$, which will result in the formula $\lambda = at + c$ where $a, c > 0$. As the value for $a$ are varied, new values will be obtained for the credit default swap premium at a specific point in time, e.g. the five year mark. The obtained values for the credit default swap premium will be plotted against the different values for $a$, and hence different values for $\lambda$, in a chart which will be used to compare market values of the credit default swap premiums with the corresponding intensities. The intensity $\lambda$ will not be a stochastic process as this will increase the complexity of the model which will not lead to a significant qualitative increase in this study.

The risk neutral probability for a default up to time $t$, $p(t)$, is given by $p(t) = 1 - e^{\int_{a}^{t} \lambda dt}$. Since $\lambda = at + c$, the following equation for the risk-neutral probability will be obtained

$$p(t) = 1 - e^{\left(\frac{at^2}{2}\right) + ct}.$$  (7)

The most common Credit default swaps have a maturity of five years and the protection premium is paid every three months. Values for the credit default swap premiums with maturity $T$ where $T=1/4k$ and $0 \leq k \leq 20$ where $k$ is an integer, will therefore be computed. To be able to compare the contingent and fee legs the first time interval has been divided into weeks and the fee leg time interval into quarters of a year. The comparison will be on a quarterly basis (as explained above) which translates into every $13^{th}$ week for the contingent leg.
For the intensity we have made 30 different computations using values for $a$ ranging from 0.001 to 0.061 and a fixed value for $c$ at 0.1. With these inputs combined with the assumptions explained above, 30 different values for the credit default swap premium will be obtained, one for each value of $a$.

\[
Q = Q(a) = \frac{(1 - 0.4)\sum_{i=0}^{M} (p(t_i) - p(t_{i-1}))e^{-0.027t_i}}{\sum_{i=1}^{n} (s_i - s_{i-1})(1 - p(s_i))e^{-0.027s_i}}
\]

Equation (8) emphasises the dependence of $a$ in $Q$. As explained in the text above, the period between premium payments, $(s_i - s_{i-1})$, is three months and $(t_i - t_{i-1})$ is one week.
6.4 Application of a Reduced Form Credit Pricing Model

In order to show the credit default premium as a function of the different parameters in the intensity, different values for the intensity has been plotted in figures 6:1 and 6:2. As can be seen, the premiums converge as time decreases. This can be explained by that the altered variable in the intensity parameter is multiplied by a time factor. Theoretical evidence points to that a credit default swap with higher default intensity should be rewarded with a higher premium. In other words, the party who sells protection for a security that is more likely to default would require a larger compensation.

Below, figure 6:1 illustrates credit default swap premiums for four different values of a; 0.001, 0.019, 0.039 and 0.059, as a function of the maturity T of the default swap. We let $0 < T < 5y$ and the step size on the time axis is three months so T is given by $T=1/4k$ for some integer $0 < k < 20$. For example, $T=5y$ implies that $T=1/4*20$ and $T=1y$ implies that $T=1/4*4$.

![Figure 6:1 Credit Default Swap Premiums 1](image)

**Figure 6:1** Credit Default Swap Premiums 1
For example, in figure 6.1 the fair premium for a two year credit default swap is given by x bp when \(a=0.001\) and y bp when \(a=0.019\). In the same way, the premium for a five year credit default swap is given by z bp when \(a=0.001\), and v bp when \(a=0.019\). The constant term in the equation is hold fixed at 0.1 for all values of \(a\). If the constant was to be changed a parallel shift in the curves would occur. A change in the constant term could be motivated by an increase or decrease in the overall risk premium.

Figure 6:2 is based on the same data as the graph above, only now 30 different values of \(a\) are included, from 0.001 to 0.059. This figure shows the sensitivity of the premium for small increases in \(a\), when \(c\) is fixed. Because of the positive time trend, the increase in \(a\) has larger effect on the premium for contracts with longer maturity, all else equal.

![Credit Default Swap Premiums 2](image)

**Figure 6:2**  
*Credit Default Swap Premiums 2*

In figure 6:3, the premium for a five year credit default swap is displayed as a function of the parameter \(a\). The relationship is almost linear. The linearity is a result of the intensity function that has been used in this study, if the intensity instead would have been described by a stochastic function, such as a poisson process, a different result may have been generated.
Theoretical models are frequently calibrated to market data in order to generate a value for the default intensity, or rather the implied default intensity. A number of things are important when a model should be calibrated against real market data, to begin with a realistic model, that well explain the features of the credit default swap premium, should be used. In order to acquire the most realistic inputs, knowledge of the market is of the essence, for example the upper and lower bounds for a five year credit default swap must be in consistence with real market data.
7. Conclusion

The most commonly used credit derivatives today are credit default swaps. The credit default swap contract provides protection against default on a reference credit, for which the protection buyer has to pay a premium which is typically paid every three months. If default occurs before maturity, the protection buyer will obtain the credit loss suffered on the reference asset.

The Credit default swaps, most frequently traded, has a maturity of five years and for Swedish banks the typical nominal amount is between five and ten million Euros for these contracts. Some 70 percent of the Swedish credit derivatives market is known names, contracts that are already traded and rated by the large credit rating firms. Other commonly used instruments are collaterised debt obligations and credit linked notes. There is extensive trade with index contracts and basket products and there is a large expected growth in this part of the credit derivatives market.

The credit derivatives market originated from banks’ need to manage their risk exposure, to certain large lenders. In Sweden however, the main use of credit derivatives is rather investment trading with the purpose to earn speculative profits. The derivatives provide a new possibility to take short positions in the corporate bond market and hence they can be treated as a complement to these.

As many credit default swaps are liquid contracts they will prove to be vital tools when it comes to measuring credit risk for banks and other market participants. The credit derivative can serve as a linkage between the transparent bond market and the more opaque market for loans and can therefore be used to determine the risk of default on a bank’s outstanding loans.

Pricing models were not used to any significant extent by the people who traded credit derivatives at Handelsbanken. Although, when a new derivative, that did not have
available market prices, was to be traded, credit ratings would be considered first. In addition internal ratings and corporate bond spreads could be used as well as advice from the bank’s credit analysts.

The basic idea of a reduced form pricing model is that default can occur at any time, as a surprise. The probability of default during a certain period is determined by the time dependent intensity parameter. In the model used in this thesis the intensity is a function of time, variable $a$ and constant $c$.

The model used in this thesis generated a positive decreasing premium function where a higher value for $a$ corresponds to a higher premium for a specific time. Even though this is a toy model, evidence points to that higher default intensity should be rewarded with a higher premium. With this in mind, the described model provides an illustration as realistic as possible under these mathematically simple conditions.

In order to calibrate this model, real market premiums can be compared to the modelled premiums to obtain an appropriate value for the intensity for a certain time to maturity. This theoretical intensity can be used to calculate premiums for credit derivatives, that are not frequently traded on the market.
8. Suggestions of Further Research

A more thorough empirical research of how Swedish banks use credit derivatives would be of interest, since the empirical results concerning the Swedish market is almost non-existent. The research conducted for this thesis has lead us to believe that credit derivatives are today used in a different manner in Sweden compared to more mature markets.

Further, a more realistic and hence complicated version of the reduced form model for pricing credit derivatives could be derived and calibrated to actual market data. For this a somewhat higher level of statistical and mathematical knowledge would be required.
9. References

9.1 Articles


9.2 Working papers


9.3 Journals

9.4 Books


9.5 Internet sources


10. Appendix

10.1 Questionnaire

Kontrakt

Vilka kontrakt handlar ni mest med? Volymer?
Säljer ni risk eller köper ni också?
Hur ser ett CDS-kontrakt ut, som ni använder?
Använder ni standardiserade kontrakt?
Hur definierar ni en kredithändelse?

Prissättning av kreditderivat

Hur estimerar ni återvärdet (recovery rate)?
Om default sker, vem bestämmer det riktiga återvärdet? (Sätter man upp en extern ”panelgrupp” som gemensamt bestämmer detta?)
Beror priset på motparten?
Använder ni historiska data?

Modeller

Vilka metoder använder ni för att prissätta de derivat som används?
Om Ni använder intensitetsbaserade modeller, vad har ni då för struktur på λ?
Använder ni marknadsdata för att kalibrera modeller?
Hur bestämmer ni intensitet och överlevnadssannolikhet?

Hedging

Hur använder ni kreditderivat för att hantera kredit risk?
Vilken kreditrisk säkras med derivat? (Låneportfölj, obligationer, enskilda företagskunder)
Vilka motparter säljer ni till? (Vilka länder, banker, försäkringsbolag…)

**Marknaden**

Vad tror ni om framtiden för den svenska kreditderivat marknaden?
(Kan kreditderivat vara ett gångbart alternativ i jakten på högre avkastning)
Vad finns det för hindrande faktorer på marknaden idag förutom problemen med prissättning?